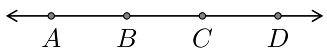
Points, Lines, and Planes

A **Point** is a position in space. A point has no length or width or thickness. A point in geometry is represented by a dot. To name a point, we usually use a (capital) letter.

A (straight) **line** has length but no width or thickness.

A line is understood to extend indefinitely to both sides. It does not have a beginning or end.



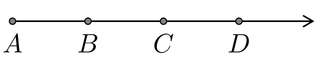
A line consists of infinitely many points. The four points A, B, C, D are all on the same line.

Postulate: Two points determine a line.

We name a line by using any two points on the line, so the above line can be named as any of the following:
$\overleftrightarrow{AB}$ $\overleftrightarrow{BC}$ $\overleftrightarrow{AC}$ $\overleftrightarrow{AD}$ $\overleftrightarrow{CD}$

Any three or more points that are on the same line are called **colinear** points. In the above, points A, B, C, D are all colinear.

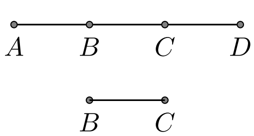
A **Ray** is part of a line that has a beginning point, and extends indefinitely to one direction.



A ray is named by using its beginning point with another point it contains.

In the above, ray $\overrightarrow{AB}$ is the same ray as $\overrightarrow{AC}$ or $\overrightarrow{AD}$. But ray $\overrightarrow{BD}$ is **not** the same ray as $\overrightarrow{AD}$.

A **(line) segment** is a finite part of a line between two points, called its **end points**. A segment has a finite length.



In the above, segment $\overline{AD}$ is not the same as segment $\overline{BC}$

Segment Addition Postulate:

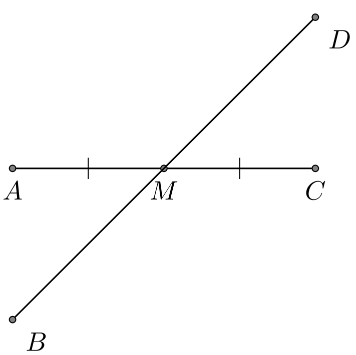
In a line segment, if points A, B, C are colinear and point B is between point A and point C, then

$\overline{AB} + \overline{BC} = \overline{AC}$

You may look at the plus sign, +, as adding the length of the segments as numbers. Or you may look at it as putting the two segments together. If two segments have the same length, we say that they are **congruent** to each other, and use the symbol, $\cong$, to denote. Pictorially, congruent segments (and other figures) can be put on top of each other (super-impose) and fit perfectly.

If two different lines intersect, then their intersection is a point, we call that point the **point of intersection** of the two lines.

If $\overline{AC}$ is a line segment and M is a point on $\overline{AC}$ that makes $\overline{AM} \cong \overline{MC}$, then M is the **midpoint** of $\overline{AC}$. If there is another segment (or line) that contains point M, that line is a **segment bisector** of $\overline{AC}$.



In the above, segment $\overline{AM} \cong \overline{MC}$ (notice that we use a tick-mark on each of the two segments to indicate that the two segments are congruent to each other), so M is the mid-point of $\overline{AC}$. Since $\overline{BD}$ is a line segment that contains the mid-point, M, of $\overline{AC}$, so $\overline{BC}$ is a **segment bisector**, or simply a **bisector** of $\overline{AC}$.

The two line segments $\overline{AC}$ and $\overline{BD}$ intersect at the point M, so M is the **point of intersection** of the two segments.

A **surface** has length and width, but no thickness.

A **plane surface**, or simply a **plane**, is a surface where any straight line that contains two points on the plane lies entirely on the plane. A plane extends indefinitely to all directions. (Think of a plane as a piece of flat paper that is infinitely large, have no thickness)

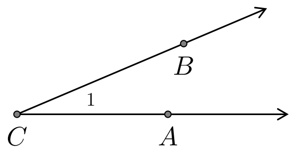
Angles

An **angle** is formed by two rays with a common end point. Each ray form a **side** of the angle and the commond end point is the **vertex** of the angle. The symbol for an angle is $\angle$

An angle can be named by naming just the point of the vertex, if there is only one angle with that vertex.

An angle can also be named using three points, where the middle point is the vertex and the first and last point is any point on each of the sides.

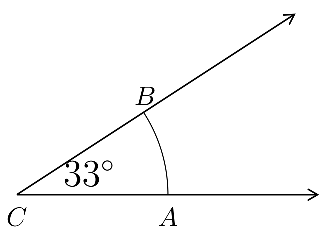
We may also put a letter or number in the angle and name the angle with using the alphabet or number.



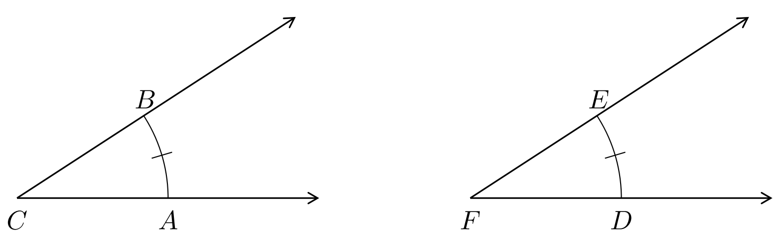
The above angle can be named as just $\angle C$, or we can name it as $\angle ACB$ or $\angle BCA$. We can also name it as $\angle 1$

The size of an angle is determined by how much one side of the angle must be rotated about the vertex to meet the other side of the angle. (Think of the size of an angle as how much the angle *opens*.)

We may also assign numbers to the size of an angle. A commonly used measurement of size of angles is **degrees**. We use a little circle to represent an angle measurement in degrees. For example, $\angle ACB$ is 33 degrees, and we write $m\angle ACB = 33^\circ$ to denote this fact.

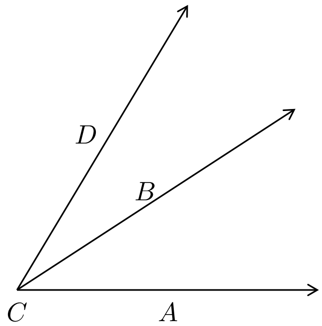


If two angles have the same size (same amount of *opening*, same degree measure), we say that the two angles are **congruent angles**.



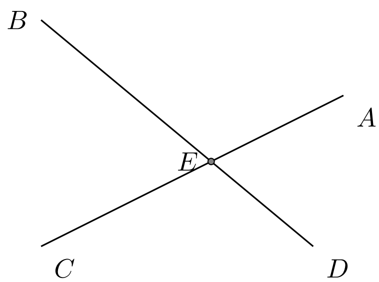
In the picture above, $\angle BCA \cong \angle EFD$. Notice that we use a tick mark on the angles to indicate that they are congruent. Geometrically, this means that the two angles can be placed on top of each other and fit perfectly.

Definition: **Adjacent Angles** are two angles that share a common side with and have the same vertex.



In the above, $\angle ACB$ and $\angle BCD$ are **adjacent angles**, they share a common side $\overline{CB}$ and have the same vertex, C.

Definition: **Vertical Angles** are two non-adjacent angles formed by two intersecting lines. Vertical angles also share the same vertex.

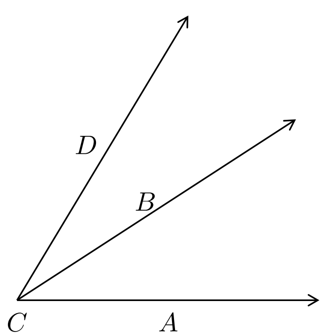


In the picture above, segment $\overline{AC}$ intersects $\overline{BD}$ at point E, so $\angle AED$ and $\angle BEC$ are **vertical angles**.

$\angle BEA$ and $\angle CED$ are also **vertical angles**.

Angle Addition Postulate:

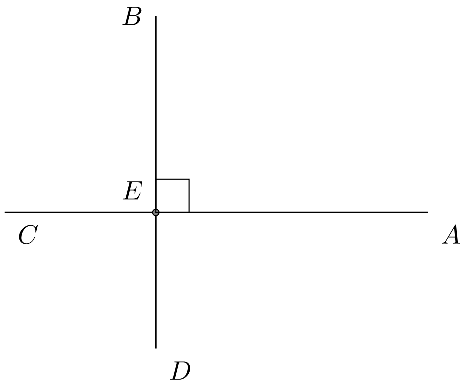
The sum of the measure of two adjacent angles is equal to the measure of the angle formed by the non-common sides of the two adjacent angles.



In the above, $m\angle ACB + m\angle BCD = m\angle ACD$.

Right Angles:

If two lines intersect and make the two adjacent angles equal to each other, then each of the equal angle is a **right angle**. The two lines that intersect this way is said to be **perpendicular** to each other.

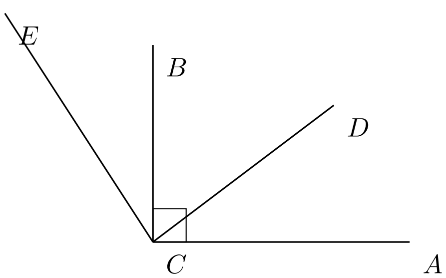


In the picture above, $\overline{CA}$ intersects $\overline{BD}$ at point E in such a way that makes $\angle CEB \cong \angle AEB$. Therefore both $\angle AEB$ and $\angle CEB$ are both **right angles**. Notice that we use an angled marking to indicate a right angle.

Since they intersect to form right angles, segments $\overline{CA}$ and $\overline{BD}$ are **perpendicular** to each other. We write $\overline{CA} \perp \overline{BD}$

An **acute angle** is one which is less than a right angle.

an **obtuse angle** is one that is greater than a right angle.



In the above, $\angle ACD$ is acute, $\angle ACB$ is right, and $\angle ACE$ is obtuse.

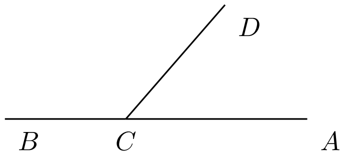
In degree measure, a right angle has a measurement of $90°$. Notice that two adjacent right angles form a straight line. Therefore, a straight line (straight angle) has a measurement of $180°$

Definition: Two angles are **complementary** if, when placed adjacent to each other with one side in common, their non-common sides form a right angle. Numerically, we say that two angles are complementary if the sum of their degree measurement equals $90°$

In the above picture, $\angle ACD$ and $\angle DCB$ are complementary because they form a right angle.

Example: A $35°$ angle is complementary to a $55°$ angle.

Definition: Two angles are **supplementary** if, when placed adjacent to each other with one side in common, their non-common sides form a straight line. Numerically, we say that two angles are supplementary if the sum of their degree measure equals $180°$

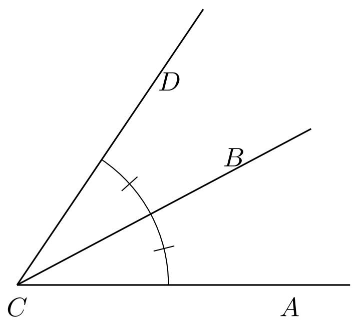


In the above, $\angle ACD$ and $\angle BCD$ are supplementary. Their non-common sides form the straight line $\overline{BA}$.

Example: A 76° angle is supplementary to a 104° angle.

Angle Bisector:

If a line cuts an angle into two equal smaller angles, the line is said to **bisect** the angle and is an **angle bisector** of the angle.



In the picture above, $\angle ACB \cong BCD$, so $\overline{CB}$ is the **angle bisector** of $\angle ACD$ Just like with segments, we use tick-marks on the angles to indicate that they are congruent to each other.

When we look at two geometrical figures as being **the same**, there are two interpretations.

Geometrically, we say the geometrical objects are **congruent**, $\cong$. This means that the two geometrical objects have the exact same shape and form and can be placed on top of each other (super-imposted) perfectly.

Algebraically, we can assign a numerical value to some of the aspects of a geometrical figure, such as the length of a line segment or a measurement to an angle. We can then interpret two geometrical objects being **equal** to mean that they have the same numerical measurement. For example, we say that $m\angle ABC = m\angle DEF$ to mean that the two angles have the same measurement (as numbers).

Under all circumstances, if two geometrical figures are congruent geometrically then their characteristics will also be equal numerically, but you should understand that there is a difference between being congruent (geometrically) and being equal (numerically).