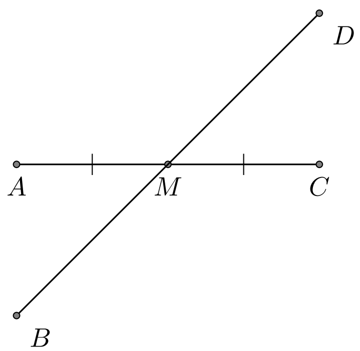
A summary of definitions, postulates, algebra rules, and theorems that are often used in geometry proofs:

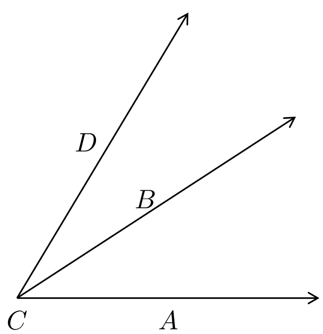
Definitions:

Definition of mid-point and segment bisector



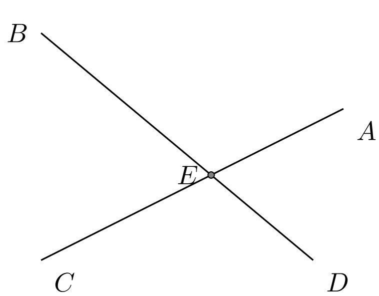
If a line $\overline{BD}$ intersects another line segment $\overline{AC}$ at a point M that makes $\overline{AM} \cong \overline{MC}$, then M is the **mid-point** of segment $\overline{AC}$, and $\overline{BD}$ is a **segment bisector** of $\overline{AC}$.

Definition of Adjacent Angles are two angles that share a common side with each other and have the same vertex.



In the above, $\angle ACB$ and $\angle BCD$ are **adjacent angles**, they share a common side $\overline{CB}$ and have the same vertex, C.

Definition of Vertical Angles are two non-adjacent angles formed by two intersecting lines. Vertical angles also share the same vertex.

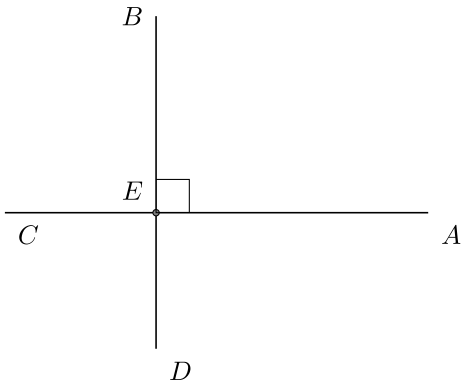


In the picture above, segment $\overline{AC}$ intersects $\overline{BD}$ at point E, so $\angle AED$ and $\angle BEC$ are **vertical angles**.

$\angle BEA$ and $\angle CED$ are also **vertical angles**.

Definition of Right Angles and Perpendicular Lines:

If two lines intersect and make the two adjacent angles equal to each other, then each of the equal angle is a **right angle**. The two lines that intersect this way is said to be **perpendicular** to each other.

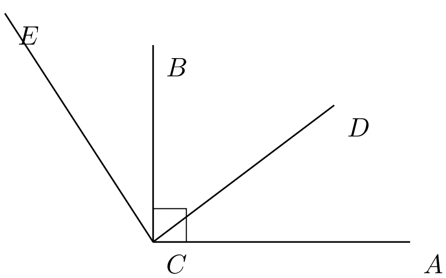


In the picture above, $\overline{CA}$ intersects $\overline{BD}$ at point E in such a way that makes $\angle CEB \cong \angle AEB$. Therefore both $\angle AEB$ and $\angle CEB$ are both **right angles**.

Since they intersect to form right angles, segments $\overline{CA}$ and $\overline{BD}$ are **perpendicular** to each other. We write $\overline{CA} \perp \overline{BD}$

An **acute angle** is one which is less than a right angle.

an **obtuse angle** is one that is greater than a right angle.



In the above, $\angle ACD$ is acute, $\angle ACB$ is right, and $\angle ACE$ is obtuse.

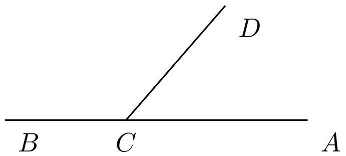
In degree measure, a right angle has a measurement of $90°$.

A straight line (straight angle) has a measurement of $180°$

Definition: Two angles are **complementary** if, when placed adjacent to each other with one side in common, their non-common sides form a right angle. Numerically, we say that two angles are complementary if the sum of their degree measurement equals $90°$

In the above picture, $\angle ACD$ and $\angle DCB$ are complementary because they form a right angle.

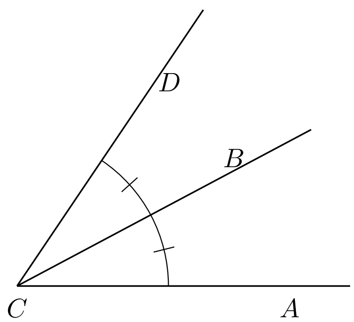
Definition: Two angles are **supplementary** if, when placed adjacent to each other with one side in common, their non-common sides form a straight line. Numerically, we say that two angles are supplementary if the sum of their degree measure equals 180°



In the above, $\angle ACD$ and $\angle BCD$ are supplementary. Their non-common sides form the straight line $\overline{BA}$.

Definition of Angle Bisector:

If a line cuts an angle into two equal smaller angles, the line is said to **bisect** the angle and is an **angle bisector** of the angle.



In the picture above, $\angle ACB \cong BCD$, so $\overline{CB}$ is the **angle bisector** of $\angle ACD$

A triangle where all three sides are unequal is a **scalene triangle**

A triangle where at least two of its sides is equal is an **isoceles triangle**

A triangle where all three sides are the same is an **equilateral triangle**.

A triangle where one of its angle is right is a **right triangle**.

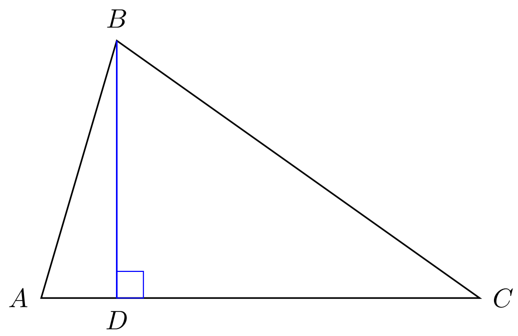
In a right-triangle, the side that is opposite the right-angle is called the **hypotenuse** of the right-triangle. The other two sides are the **legs** of the right-triangle.

A triangle where one of its angle is obtuse is an **obtuse triangle**:

A triangle that does not have any obtuse angle (all three angles are acute) is called an **acute triangle**.

Altitude of a Triangle

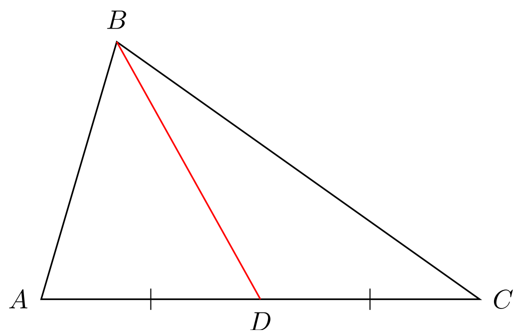
In a triangle, if through any vertex of the triangle we draw a line that is perpendicular to the side opposite the vertex, this line is an **altitude** of the triangle. The line opposite the vertex where the altitude is perpendicular to is the **base**.



In $\triangle ABC$ above, $\overline{BD}$ is an altitude. It contains vertex B and is perpendicular to $\overline{AC}$, which is the base.

Median of a Triangle:

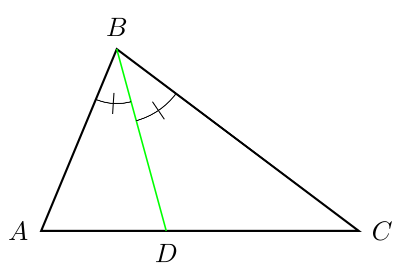
In any triangle, if through one of its vertex we draw a line that **bisects** the opposite side, this line is called a **median** of the triangle.



In $\triangle ABC$ above, $\overline{BD}$ bisects $\overline{AC}$ in D ($\overline{AD} \cong \overline{DC}$), so by definition, $\overline{BD}$ is a median of $\triangle ABC$

Angle Bisector

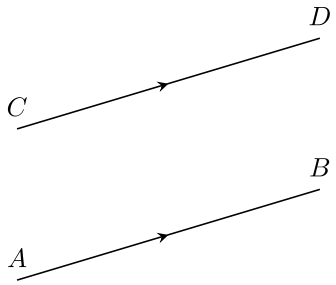
An **angle bisector** of a triangle is a line that bisects an angle of the triangle and intersects the opposite side.



In $\triangle ABC$ above, $\overline{BD}$ is an angle bisector of $\angle ABC$

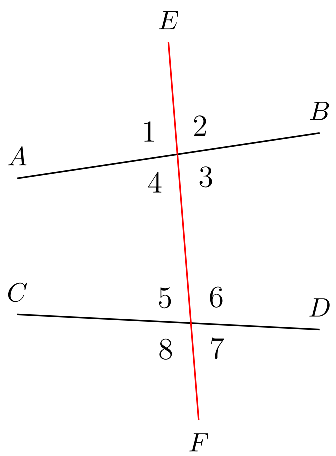
Parallel Lines:

Definition: We say that two lines (on the same plane) are **parallel** to each other if they never intersect each other, ragardless of how far they are extended on either side. Pictorially, parallel lines run along each other like the tracks of a train.



Lines $\overline{AB}$ and $\overline{CD}$ are parallel to each other. We use the symbol $||$ to represent two lines being parallel. We write $\overline{AB}||\overline{CD}$ to denote $\overline{AB}$ is parallel to $\overline{CD}$. We use little arrows on the two lines to indicate that they are parallel to each other.

A **transversal** of two (or more) lines is another line that intersects the two lines.



In the picture above, line $\overline{EF}$ is a transversal of lines $\overline{AB}$ and $\overline{CD}$. It intersects the two lines and forms 8 angles with the two lines. We name the relationship of the angle pairs based on their position with respect to each other and to the lines $\overline{AB}$ and $\overline{CD}$

The angles $\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$ are **exterior** angles because they are on the *outside* of lines $\overline{AB}$ and $\overline{CD}$.

The angles $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$ are **interior** angles because they are on the *inside* of lines $\overline{AB}$ and $\overline{CD}$.

Corresponding Angles are angles that are on the same side of the transversal and on the same side of each intersected line. In the picture above,

$\angle 2$ and $\angle 6$ are corresponding angles.

$\angle 3$ and $\angle 7$ are corresponding angles.

$\angle 1$ and $\angle 5$ are corresponding angles.

$\angle 4$ and $\angle 8$ are corresponding angles.

Alternate Interior Angles are interial angles on opposite sides of the transversal.

$\angle 3$ and $\angle 5$ are alternate interior angles.

$\angle 4$ and $\angle 6$ are alternate interior angles.

Alternate Exterior Angles are exterior angles on opposite sides of the transversal.

$\angle 2$ and $\angle 8$ are alternate exterior angles.

$\angle 1$ and $\angle 7$ are alternate exterior angles.

Same-Side Interior Angles are interior angles on the same side of the transversal.

$\angle 4$ and $\angle 5$ are same-side interior angles.

$\angle 3$ and $\angle 6$ are same-side interior angles.

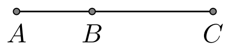
Same-Side Exterior Angles are exterior angles on the same side of the transversal.

$\angle 2$ and $\angle 7$ are same-side exterior angles.

$\angle 1$ and $\angle 8$ are same-side exterior angles.

Properties, Postulates, Theorems:

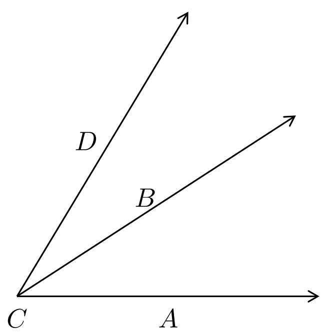
Segment Addition Postulate:



In a line segment, if points A, B, C are colinear and point B is between point A and point C, then: $\overline{AB} + \overline{BC} = \overline{AC}$

Angle Addition Postulate:

The sum of the measure of two adjacent angles is equal to the measure of the angle formed by the non-common sides of the two adjacent angles.



In the above, $m\angle ACB + m\angle BCD = m\angle ACD$.

Properties of Equality:

For any object x, $x = x$ (**reflexive property**).

If $a = b$, then $b = a$ (**symmetric property**)

If $a = b$, and $b = c$, then $a = c$ (**transitive property**)

If $a = b$, then anywhere a is used in a statement, b can be used instead and the meaning of the statement is unchanged. (**substitution property**)

If $a = b$ and $c = d$, then $a + c = b + d$ (**addition postulate**)

If $a = b$ and $c = d$, then $a - c = b - d$ (**subtraction postulate**)

Complementary Angle Theorem: If two angles are complementary to the same angle, then they are congruent to each other

Supplementary Angle Theorem: If two angles are supplementary to the same angle, then they are congruent to each other

Vertical Angles Theorem: Vertical Angles are Congruent.

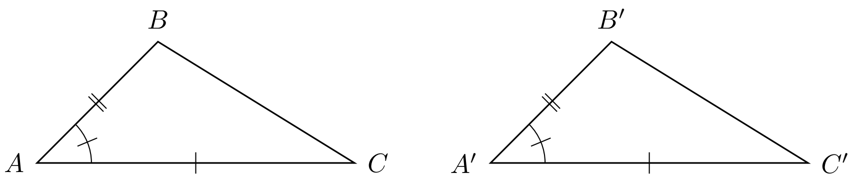
Ways to prove triangles are congruent:

Side-Side-Side (SSS)

If all three sides of a triangle is congruent to all three sides of another triangle, the two triangles are congruent.

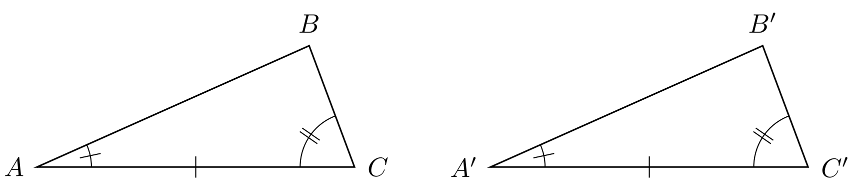
Side-Angle-Side (SAS):

If two sides of a triange is congruent to two sides of another triangle, and the angle formed by the two sides is also congruent, then the two triangles are congruent.



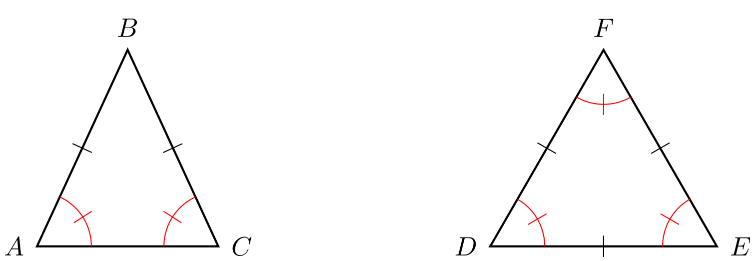
Angle-Side-Angle (ASA):

If two angles of a triange is congruent to two angles of another triangle, and the side between the two angles is also congruent, then the two triangles are congruent.

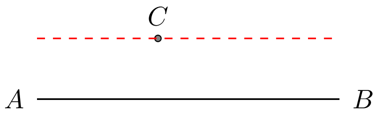


Isoceles Triangle Theorem: In an isoceles triangle, the **base angles** (the angles on the opposite sides of the congruent sides) are congruent.

Equilateral Triangle Theorem: In an equilateral triangle, all three angles are congruent.



Parallel Postulate: Given a line and given a point not on the line, there is one and only one line that can be drawn that contains the given point and is parallel to the given line.



Theorem: Corresponding Angles: If two lines are cut by a transversal that makes a pair of corresponding angles congruent, then the two lines are parallel.

The converse of this theorem is also true:

If two parallel lines are cut by a transversal, then their corresponding angles are congruent to each other

Alternate Interior Angles: If two lines are cut by a transversal that make a pair of alternate interior angles congruent to each other, then the two lines are parallel.

The converse of this theorem is also true:

If two parallel lines are cut by a transversal, then the alternate interior angles are congruent

Alternate Exterior Angles: If two lines are cut by a transversal that make a pair of alternate exterior angles congruent to each other, then the two lines are parallel.

The converse of this theorem is also true:

If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent

Same-Side Interior Angles: If two lines are cut by a transversal that make a pair of same-side interior angles supplementary to each other, then the two lines are parallel.

The converse of this theorem is also true:

If two parallel lines are cut by a transversal, then the same-side interior angles are supplementary

Same-Side Exterior Angles: If two lines are cut by a transversal that make a pair of same-side interior angles supplementary to each other, then the two lines are parallel.

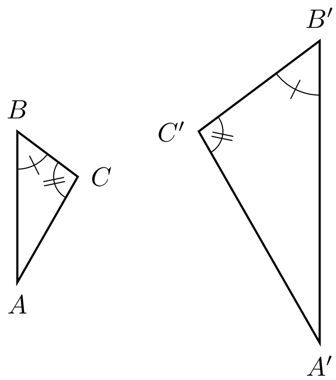
The converse of this theorem is also true:

If two parallel lines are cut by a transversal, then the same-side exterior angles are supplementary

Theorem: Sum of Interior Angles of a Triangle: The sum of the three interior angles of any triangle is always equal to $\mathbf{180^\circ}$ (two right angles)

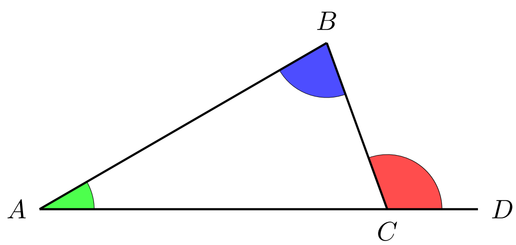
If two angles of one triangle is congruent to two angles of another

triangle, then the third angle must also be congruent

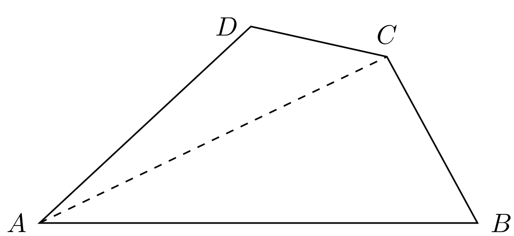


Theorem: The sum of any two interior angles of a triangle is equal to the opposite exterior angle.

In the picture below, $m\angle A + m\angle B = m\angle BCD$



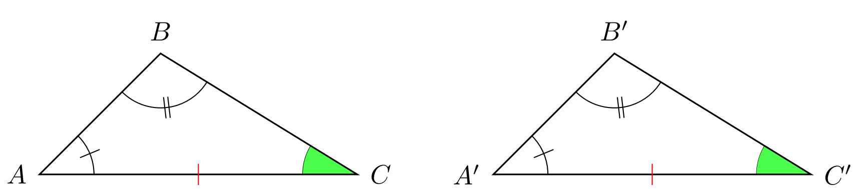
Theorem: Sum of Interior Angles of a Quadrilateral: The sum of the interior angles of a quadrilateral is 360°



In picture above, notice that $\overline{AC}$ divides quadrilateral $ABCD$ into two triangles, $\triangle ACD$ and $\triangle ACB$. The sum of the interior angles of each triangle is 180°, so the sum of the interior angles of a quadrilateral is 360°

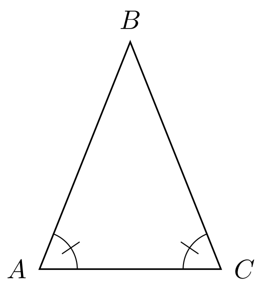
In general, for a polygon with n sides, the sum of its interior angles is equal to $(n-2)180°$

Angle-Angle-Side (AAS): If two angles of a triangle is congruent to two angles of another triangle, and a corresponding side (not necessarily between the two angles) is also congruent, then the two triangles are congruent.



Converse of the isoceles triangle theorem:

In a triangle if two angles are congruent, then the two sides opposite the two angles are also congruent



Theorem: If a triangle has all three interior angles congruent, then the triangle is equilateral

Hypotenuse-Leg (HL), right-triangle only:

If the hypotenuse of two right-triangles is congruent and one of their legs is congruent, then the two right-triangles are congruent.