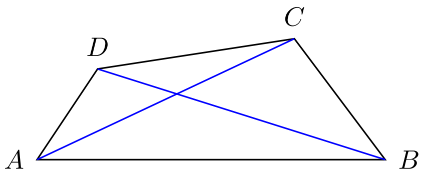
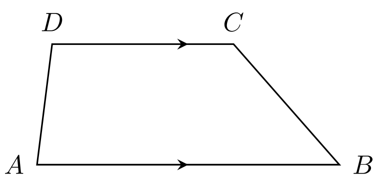
Definition: A **quadrilateral** is a polygon with 4 sides. A **diagonal** of a quadrilateral is a line segment whose end-points are opposite vertices of the quadrilateral. In picture below, $ABCD$ is a quadrilateral, $\overline{AC}$, $\overline{BD}$ are the two diagonals.



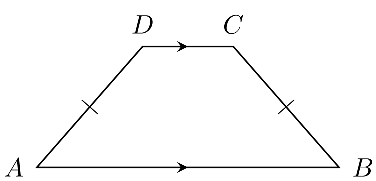
We name a quadrilateral by naming the four vertices in consecutive order. So we can name the quadrilateral as $ABCD$, or quadrilateral $BCDA$, or $ADCB$.

Definition: A **Trapezoid** is a quadrilateral with a pair of parallel sides.



The pair of parallel sides ($\overline{AB}||\overline{DC}$) are called the **bases** of the trapazoid, and the non-parallel sides ($\overline{DA}, \overline{CB}$) form the **legs** of the trapazoid.

If the two legs of the trapazoid are congruent to each other, then we have an **isoceles trapazoid**.

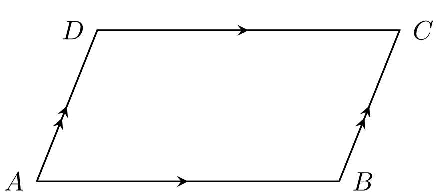


Theorem: The base angles of an isoceles trapazoid are congruent.

In the above isoceles trapazoid, $\angle A \cong \angle B$

The converse of this statement is also true: If the base angles of a trapazoid is congruent, then the trapazoid is isoceles.

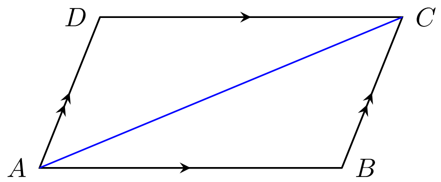
Definition: A **parallelogram** is a quadrilateral where **both** pairs of opposite sides are parallel. We use the symbol ▱ to represent a parallelogram.



In ▱ $ABCD$, $\overline{AB}||\overline{DC}$, $\overline{AD}||\overline{BC}$.

Theorem: Opposite sides of a parallelogram are congruent.

Proof: Given $\square ABCD$, we must prove that $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$. We contruct the diagonal, $\overline{AC}$, of the parallelogram.

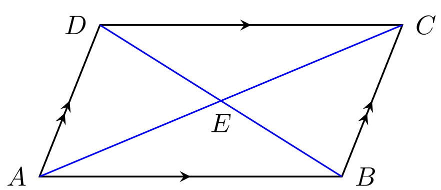


	Statements		Reasons
1.	$\overline{AC}$ is a diagonal to $\square ABCD$	1.	Given
2.	$\overline{AB} \| \overline{DC}$, $\overline{AD} \| \overline{BC}$	2.	Def. of $\square$
3.	$\angle DCA \cong BAC$, $\angle DAC \cong \angle BCA$	3.	Alternate Interior angles
4.	$\overline{AC} \cong \overline{AC}$	4.	Reflexive
5.	$\triangle CAB \cong \triangle ACD$	5.	ASA
6.	$\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$	6.	CPCTC

The converse of this statement is also true. That is, if both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Theorem: The diagonals of a parallelogram bisect each other.

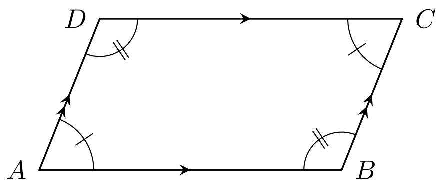
Proof: Given $\square ABCD$, let the diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E, we must prove that $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$.



	Statements		Reasons
1.	$\overline{AC}$ and $\overline{BD}$ are diagonals to $\square ABCD$	1.	Given
2.	$\overline{AB} \| \overline{DC}$, $\overline{AD} \| \overline{BC}$	2.	Def. of $\square$
3.	$\angle DCE \cong BAE$, $\angle CDE \cong \angle ABE$	3.	Alternate Interior angles
4.	$\overline{DC} \cong \overline{AB}$	4.	opposite sides of $\square$ are $\cong$
5.	$\triangle ABE \cong \triangle CDE$	5.	ASA
6.	$\overline{AE} \cong \overline{CE}$, $\overline{BE} \cong \overline{DE}$	6.	CPCTC
7.	$\overline{AC}$, $\overline{BD}$ bisect each other	7.	Def. of segment bisector

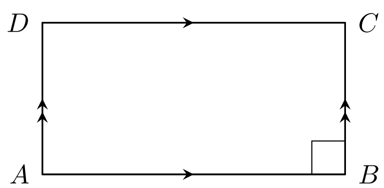
The converse is also true: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Theorem: Opposite angles of a parallelogram are congruent to each other. In $\square ABCD$, $\angle A \cong \angle C$, and $\angle B \cong \angle D$.



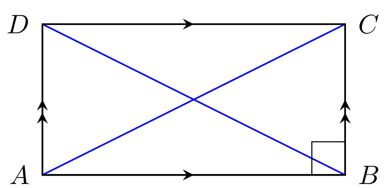
Conversely, if both pairs of opposite angles of a quadrilateral are congruent to each other, then the quadrilateral is a parallelogram.

A **rectangle** is a parallelogram with all four angles being right angles. In a parallelogram, if one angle is a right angle, then all four angles are right (why?).



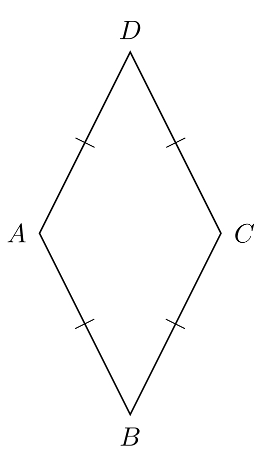
$ABCD$ is a rectangle.

Theorem: The two diagonals of a rectangle are congruent.



In rectangle $ABCD$, $\overline{AC} \cong \overline{BD}$.

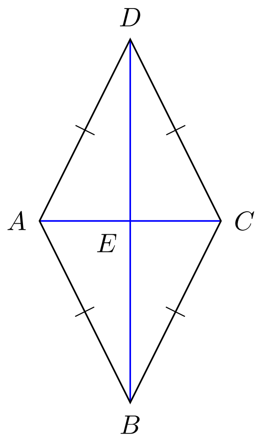
A **rhombus** is a parallelogram with all four sides congruent to each other.



$ABCD$ is a rhombus, which means $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$. A rhombus has a diamond-like shape.

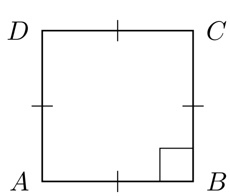
Theorem: The diagonals of a rhombus are perpendicular to each other.

Proof: Given rhombus $ABCD$, let the diagonals $\overline{AC}$, $\overline{BD}$ intersect at E, we must prove that $\overline{AC} \perp \overline{BD}$



Statements		Reasons	
1.	$\overline{AC}$ and $\overline{BD}$ are diagonals to rhombus $ABCD$	1.	Given
2.	$\overline{AB} \cong \overline{BC}$	2.	Def. of rhombus
3.	$\overline{AE} \cong \overline{CE}$	3.	Diagonals of ▱ bisect each other
4.	$\overline{BE} \cong \overline{BE}$	4.	Reflexive
5.	$\triangle BAE \cong \triangle BCE$	5.	SSS
6.	$\angle BEA \cong \angle BEC$	6.	CPCTC
7.	$\overline{AC} \perp \overline{BD}$	7.	Def. of perpendicular lines

A **square** is a parallelogram with four congruent sides and four right angles. In other words, a square is a rectangle **and** a rhombus.



$ABCD$ is a square, which means that $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are all right angles. In addition, $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$

Understand that rectangles, rhombus, squares are all parallelograms. Therefore they all have properties that a parallelogram has. Any theorem that is true about a parallelogram can be applied to a rectangle, rhombus, or square. These special parallelograms, of course, have more specific properties that may not be shared by other parallelograms. We use a table to indicate the properties that are true for each kind of figure:

Properties	Parallelogram	Rectangle	Rhombus	Square
Opposite sides Parallel	yes	yes	yes	yes
Opposite sides Congruent	yes	yes	yes	yes
Diagonals bisect each other	yes	yes	yes	yes
Opposite angles are congruent	yes	yes	yes	yes
Diagonals are congruent	no	yes	no	yes
All four angles are right	no	yes	no	yes
Diagonals are perpendicular	no	no	yes	yes
All four sides congruent	no	no	yes	yes

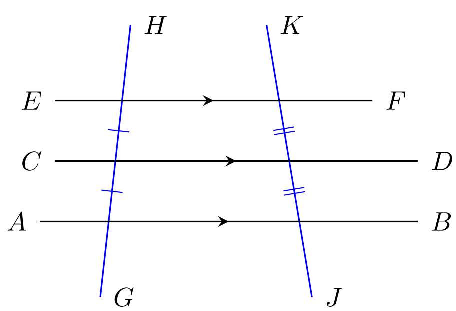
To prove that a parallelogram is a rectangle, we need to prove that one of its interior angle is right. We can also try to prove that its diagonals are congruent.

To prove that a parallelogram is a rhombus, we need to prove that its four sides are congruent. We can also try to prove that its diagonals are perpendicular.

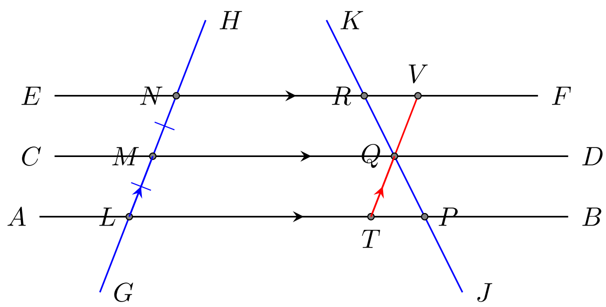
To prove that a parallelogram is a square, we need to prove that it is a rectangle and a rhombus.

Theorem: If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on all other transversals.

In picture below, $\overline{AB}||\overline{CD}||\overline{EF}$. If $\overline{HG}$ is a transversal cutoff into equal parts by the three parallel lines, then $\overline{KJ}$ will also be cut-off into equal parts by the three parallel lines.



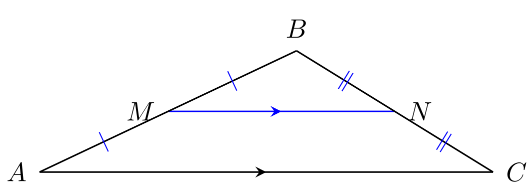
Proof: In the picture below, given lines $\overline{AB}||\overline{CD}||\overline{EF}$, and $\overline{LM} \cong \overline{MN}$, we need to prove that $\overline{RQ} \cong \overline{PQ}$. We will do so by introducing a new line, the line through Q parallel to $\overline{HG}$.



Statements	Reasons
1. $\overline{AB} \| \| \overline{CD} \| \| \overline{EF}$, $\overline{LM} \cong \overline{NM}$	1. Given
2. Construct $\overline{VT}$ through Q parallel to $\overline{LN}$	2. Parallel Postulate
3. $NMQV$ and $MLTQ$ are parallelograms	3. Def. of Parallelograms
4. $\overline{MN} \cong \overline{QV}$, $\overline{LM} \cong \overline{TQ}$	4. Opposite sides of ▱ are $\cong$
5. $\overline{VQ} \cong \overline{TQ}$	5. Substitution
6. $\angle RVQ \cong \angle PTQ$, $\angle VRQ \cong \angle TPQ$	6. Alternate Interior Angles
7. $\triangle RVQ \cong \triangle PTQ$	7. AAS
8. $\overline{VQ} \cong \overline{TQ}$	8. CPCTC

Theorem: If a line is drawn from the midpoint of one side of a triangle and parallel to a second side, then that line bisects the third side.

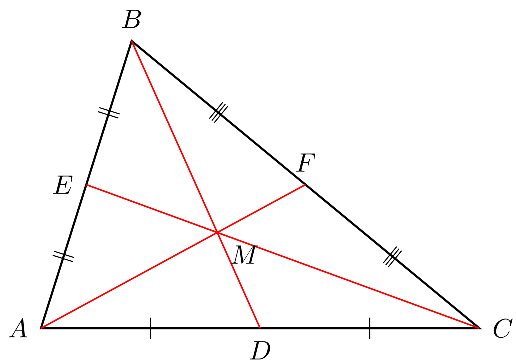
In picture below, M is the midpoint of $\overline{AB}$. If we construct a line through M parallel to $\overline{AC}$, then this line will intersect $\overline{BC}$ at N, where N is the midpoint of $\overline{BC}$



The converse of this theorem is also true. If a line connects the midpoints of two sides of a triangle, then the line is parallel to the third side. In addition, the length of this line is half of the length of the third side.

In the picture above, if M is the midpoint of $\overline{AB}$ and N is the midpoint of $\overline{CB}$, then $\overline{MN} || \overline{AC}$, and $\overline{MN} = \dfrac{1}{2}\overline{AC}$

Theorem: The three medians of a triangle intersect at a point (the **centroid** of the triangle). This point is two-thirds of the distance from any vertex to the midpoint of the opposite side.



In the above, if $\overline{AF}$, $\overline{CE}$, and $\overline{BD}$ are medians of $\triangle ABC$, then they intersect at a single point, M, and $\overline{CM} = 2\overline{ME}$, $\overline{AM} = 2\overline{MF}$, $\overline{BM} = 2\overline{MD}$.