1. Write the complex numbers in polar form:

a. $1+i$

Ans: $\sqrt{2}\left[\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right]$

b. $-1+i$

Ans: $\sqrt{2}\left[\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)\right]$

c. $1-\sqrt{3}i$

Ans: $2\left[\cos\left(\frac{5\pi}{3}\right)+i\sin\left(\frac{5\pi}{3}\right)\right]$

d. $4-4i$

Ans: $4\sqrt{2}\left[\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)\right]$

e. -2

Ans: $2\left[\cos\left(\pi\right)+i\sin\left(\pi\right)\right]$

2. Write the following complex number in rectangular form:

a. $2\left[\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right]$

Ans: $-1+\sqrt{3}i$

b. $4\left[\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)\right]$

Ans: $2\sqrt{2}-2\sqrt{2}i$

c. $2\left[\cos\left(\frac{5\pi}{6}\right)+i\sin\left(\frac{5\pi}{6}\right)\right]$

Ans: $-\sqrt{3}+i$

d. $3\left[\cos\left(\frac{3\pi}{2}\right)+i\sin\left(\frac{3\pi}{2}\right)\right]$

Ans: $-3i$

3. Simplify the expression and write your answer in standard form $a+bi$

a. $\left[4\left(\cos\left(\frac{2\pi}{9}\right)+i\sin\left(\frac{2\pi}{9}\right)\right)\right]^3$

Ans: $-32+32\sqrt{3}i$

b. $\left[2\left(\cos\left(\frac{\pi}{10}\right)+i\sin\left(\frac{\pi}{10}\right)\right)\right]^5$

Ans: $32i$

c. $\left[\sqrt{2}\left(\cos\left(\frac{5\pi}{16}\right)+i\sin\left(\frac{5\pi}{16}\right)\right)\right]^4$

Ans: $-2\sqrt{2}-2\sqrt{2}i$

d. $\left[\sqrt{5}\left(\cos\left(\frac{3\pi}{16}\right)+i\sin\left(\frac{3\pi}{16}\right)\right)\right]^4$

Ans: $-\frac{25\sqrt{2}}{2}+\frac{25\sqrt{2}}{2}i$

e. $\left[\sqrt{3}\left(\cos\left(\frac{5\pi}{18}\right)+i\sin\left(\frac{5\pi}{18}\right)\right)\right]^6$

Ans: $\frac{27}{2}-\frac{27\sqrt{3}}{2}i$

f. $(1-i)^5$

Ans: $-4+4i$

g. $\left(\sqrt{3}-i\right)^6$

Ans: -64

4. Find all cube roots of $1+i$.

Ans:

$z_0=\sqrt[6]{2}\left[\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)\right]$

$z_1=\sqrt[6]{2}\left[\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)\right]$

$z_2=\sqrt[6]{2}\left[\cos\left(\frac{17\pi}{12}\right)+i\sin\left(\frac{17\pi}{12}\right)\right]$

5. Find all fourth roots of $\sqrt{3}-i$.

Ans:

$z_0=\sqrt[4]{2}\left[\cos\left(\frac{11\pi}{24}\right)+i\sin\left(\frac{11\pi}{24}\right)\right]$

$z_1=\sqrt[4]{2}\left[\cos\left(\frac{23\pi}{24}\right)+i\sin\left(\frac{23\pi}{24}\right)\right]$

$z_2=\sqrt[4]{2}\left[\cos\left(\frac{35\pi}{24}\right)+i\sin\left(\frac{35\pi}{24}\right)\right]$

$z_3=\sqrt[4]{2}\left[\cos\left(\frac{47\pi}{24}\right)+i\sin\left(\frac{47\pi}{24}\right)\right]$

6. Find all fourth roots of $4-4\sqrt{3}i$.

Ans:

$$z_0 = \sqrt[4]{8}\left[\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right]$$

$$z_1 = \sqrt[4]{8}\left[\cos\left(\frac{11\pi}{12}\right) + i\sin\left(\frac{11\pi}{12}\right)\right]$$

$$z_2 = \sqrt[4]{8}\left[\cos\left(\frac{17\pi}{12}\right) + i\sin\left(\frac{17\pi}{12}\right)\right]$$

$$z_3 = \sqrt[4]{8}\left[\cos\left(\frac{23\pi}{12}\right) + i\sin\left(\frac{23\pi}{12}\right)\right]$$

7. Find all the fifth roots of i

Ans:

$$z_0 = \cos\left(\frac{\pi}{10}\right) + i\sin\left(\frac{\pi}{10}\right)$$

$$z_1 = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)$$

$$z_2 = \cos\left(\frac{9\pi}{10}\right) + i\sin\left(\frac{9\pi}{10}\right)$$

$$z_3 = \cos\left(\frac{13\pi}{10}\right) + i\sin\left(\frac{13\pi}{10}\right)$$

$$z_4 = \cos\left(\frac{17\pi}{10}\right) + i\sin\left(\frac{17\pi}{10}\right)$$

8. Find all the fourth roots of unity.

Ans: $1,\ i,\ -1,\ -i$

9. Find all the sixth roots of unity.

Ans: $1, \quad \frac{1}{2} + \frac{\sqrt{3}}{2}i, \quad -\frac{1}{2} + \frac{\sqrt{3}}{2}i, \quad -1, \quad -\frac{1}{2} - \frac{\sqrt{3}}{2}i, \quad \frac{1}{2} - \frac{\sqrt{3}}{2}i,$