General Differential Equations
A **differential equation** is an equation that contains an unknown function and one or more of its derivatives.
A function f(x) is called a **solution** of a differential equation if the equation is satisfied (true) when y = f(x) and its derivatives are substituted into the equation.
A **general solution** to a differential equation is a family of functions, f(x) + C that satisfies the equation. A **particular solution** to a differential equation is a solution, satisfying an initial condition, or initial value $y(x_0) = y_0$.

Separable Differential Equations
A **separable equation** is a first-order differential equation in which the expression for $\frac{dy}{dx}$ can be factored as a function of x times a function of y, in other words written in the form $\frac{dy}{dx} = \boldsymbol{g(x)f(y)}$.
Process to solve a separable differential equation: $\frac{dy}{dx} = g(x)f(y)$ 1 - Separate the variables: $\frac{dy}{f(y)} = g(x)dx$ 2 - Integrate both sides: $\int \frac{dy}{f(y)} = \int g(x)dx$ + C (Add "+C" on the right side of the above integration.) This is the implicit form of the general solution to the diff. equation. 3 - Solve for y, if possible, to get the explicit solution y = F(x) + C. 4 - Find the particular solution if you are given initial values $y(x_0) = y_0$.

Linear Differential Equations

A **linear differential equation** is one that can be put into the form

$$\frac{dy}{dx} + P(x) \cdot y = Q(x) \quad \textit{where P and Q are continuous functions of x on some interval.}$$

Process to solve a linear differential equation:

1 - Put in standard form $\frac{dy}{dx} + P(x) \cdot y = Q(x)$

2 - Evaluate the Integrating factor $I(x) = e^{\int P(x)dx}$ Do not add "+C" and simplify expression as much as possible.

3 - Multiply both sides of equation by I(x)

$$I(x)\left[\frac{dy}{dx} + P(x) \cdot y\right] = I(x)Q(x)$$

4 - Transform the left side of the question to derivative of I(x)y , and simplify the right side:

$$\frac{d}{dx}[I(x) \cdot y] = I(x)Q(x)$$

5 - Integrate both sides, add "+C" on right side:

$$I(x) \cdot y = \int I(x)Q(x)dx + C \ \ (\textit{evaluate the integral, for the step below, suppose} \int I(x)Q(x)dx = F(x))$$

6 - Solve for y to get the explicit general solution of the equation:

$$y = \frac{F(x) + C}{I(x)}$$

7 - If asked, find the particular solution by substituting initial values $y(x_0) = y_0$ into equation, solving for C, and rewriting general solution with the calculated value of C.

Mixing Problems	
Given an initial volume of liquid (used to find concentration OUT) & an initial concentration of a substance (e.g. salt). **Initial concentration is initial value: y(0) = inital amt. of "salt."**	Given Rate IN of liquid & concentration of liquid IN. Also given Rate OUT of liquid. Assume tank is thoroughly mixed. **Concentration OUT =** $\boldsymbol{\frac{y}{initial\ volume + (rate\ IN)\cdot t - (rate\ OUT)\cdot t}}$
Let $y(t) = \textit{amount of substance in tank at time } t$.	$\frac{dy}{dt} = \textit{Rate of Substance IN} - \textit{Rate of Substance OUT}$
Rate of Substance IN = (Rate IN of liquid)(Concentration IN)	Rate of Substance OUT = (Rate OUT of liquid)(Concentration OUT)

1- Set up the differential equation using equation in above second column, second row.
2- Use separable or linear processes to find the general solution to the differential equation (solve for y(t))
3- Use the initial value y(0) = initial amount of the substance in the tank at t = 0 to find the particular solution (sub values, solve for C).
4- Answer the question, "How much of the substance is in the tank after "a" minutes?" Answer = y(a).

Parametric Equations

x = f(t) y = g(t) on $a \le t \le b$ has intitial point $(f(a), g(a))$ and terminal point $(f(b), g(b))$.

To **graph a set of parametric equations,** build a table with columns for t, x=f(t), y=g(t), and ultimate point to graph (x,y).
In the table, list the values of t in order from smallest to biggest.
Plot the points (x,y) in the x-y plane, then connect the dots in order of t, from smallest to biggest.
Add arrows on the graph in the direction that the graph should be drawn -- in order of t, from smallest to biggest.

Eliminate the Parameter to find a Cartesian equation of the curve.

Option 1	Option 2: Trigonometric equations.	Option 3 (Innovative Substitution)
• Solve one of the equations for the parameter (e.g. parameter t). • Substitute this value of the parameter into the other equation.	• Solve each equation for the trigonometric function it contains. • Identify a fundamental trig identity that contains the trig functions in the equations. • Substitute the values of the trig functions from the parametric equations into the fundamental trig identity. • Solve for y or put equation into a standard form to identify its graph shape.	Recognize when one parametric equation's expression can easily be substituted into the other equation.
$cos^2 t + sin^2 t = 1$	$tan^2 t + 1 = sec^2 t$	$cot^2 t + 1 = csc^2 t$

Derivatives of Parametric Equations x = f(t) y = g(t)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Equation of Tangent Line at t = a

To find slope 'm' at t = a:
Find $\frac{dy}{dx}$ and evaluate at t = a.

To find the point of the tangent line at t = a
Evaluate x = f(t), y = g(t) at t = a.
$(f(a), g(a)) = (x_1, y_1)$

Equation of tangent line at t = a : $y - y_1 = m(x - x_1)$

Surface Area of Revolution for x = f(t) and y = g(t)

About the x-axis

$$\Delta SA = 2\pi y ds = 2\pi g(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

About the y-axis

$$\Delta SA = 2\pi x ds = 2\pi f(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Polar Coordinates

To convert from (x,y) to (r, θ)
- $r = \sqrt{x^2 + y^2}$
- $\theta = tan^{-1}\left(\frac{y}{x}\right)$ *only if* (x, y) *is in QI or QIV*
- $\theta = \pi + tan^{-1}\left(\frac{y}{x}\right)$ *only if* (x, y) *is in QII or QIII*

To convert from (r, θ) to (x,y)
- $x = r cos\theta$
- $y = r sin\theta$

Tangents to Polar Curves r = f(θ), x = rcosθ y = rsinθ ; Find slope of tangent at θ = a. Substitute value of θ into equation.

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta} sin\theta + r cos\theta}{\frac{dr}{d\theta} cos\theta - r sin\theta}$$

Area of Region defined by Polar Equation r = f(θ) from $a \le \theta \le b$

$$A = \int_a^b \frac{1}{2} r^2 d\theta$$

Arc Length of Curve defined by polar function r = f(θ), $a \le \theta \le b$

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Exponential Properties

$e^{lnx} = x$	$aln(x) = ln(x^a)$	$lnx + lny = \ln(x \cdot y)$ $lnx - lny = ln\left(\frac{x}{y}\right)$

Trigonometric Values

For θ= π/4, cosθ = sinθ = $1/\sqrt{2}$, tanθ = cotθ = 1

For θ = $\pi/6$, sinθ = 1/2, cosθ = $\sqrt{3}/2$, For θ = $\pi/3$, sinθ = $\sqrt{3}/2$, cosθ = 1/2