Ch 8

Find the exact area of the surface obtained by rotating the curve about the x-axis.

8. $y = \sqrt{1+e^x} = (1+e^x)^{1/2}$ $\quad 0 \le x \le 1$

$$\frac{dy}{dx} = \frac{1}{2}(1+e^x)^{-1/2} \cdot e^x = \frac{e^x}{2\sqrt{1+e^x}}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{e^{2x}}{4(1+e^x)}$$

$$1+\left(\frac{dy}{dx}\right)^2 = \frac{4(1+e^x)+e^{2x}}{4(1+e^x)} = \frac{4+4e^x+e^{2x}}{4(1+e^x)}$$

$$= \frac{(e^x+2)^2}{4(1+e^x)}$$

$$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \frac{e^x+2}{2\sqrt{1+e^x}}$$

$$\Delta SA = 2\pi r\, ds$$

$$= 2\pi f(x) \cdot ds$$

$$= \cancel{2}\pi \cancel{\sqrt{1+e^x}} \cdot \frac{e^x+2}{\cancel{2}\cancel{\sqrt{1+e^x}}}$$

$$= \pi(e^x+2)$$

$$SA = \pi \int_0^1 (e^x+2)\,dx$$

$$= \pi(e^x+2x)\Big|_0^1 = \pi(e^1+2(1)) - \pi(e^0+2(0))$$

$$= \pi e + 2\pi - \pi$$

$$= \pi e + \pi = \boxed{\pi(e+1)}$$