

7.4

$$(18) \int \frac{x^2+2x-1}{x^3-x}$$

$$\frac{x^2+2x-1}{x^3-x} = \frac{x^2+2x-1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$x^2+2x-1 = A(x-1)(x+1) + Bx(x+1) + Cx(x-1)$$

$$x=1 \quad 1^2+2(1)-1 = A(0)(2) + B(1)(2) + C(1)(0)$$

$$2 = 2B \quad \boxed{B=1}$$

$$x^2+2x-1 = A(x-1)(x+1) + x(x+1) + Cx(x-1)$$

$$x=0$$

$$-1 = A(-1)(1) + 0(1) + C(0)(-1)$$

$$-1 = -A \quad \boxed{A=1}$$

$$x^2+2x-1 = (x-1)(x+1) + x(x+1) + Cx(x-1)$$

$$x=-1$$

$$(-1)^2+2(-1)-1 = (-2)(0) + -1(0) + C(-1)(-2)$$

$$1-2-1 = 2C$$

$$-2 = 2C \quad \boxed{C=-1}$$

$$\frac{x^2+2x-1}{x^3-x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} = \frac{1}{x} + \frac{1}{x-1} - \frac{1}{x+1}$$

$$\int \frac{x^2+2x-1}{x^3-x} dx = \int \left(\frac{1}{x} + \frac{1}{x-1} - \frac{1}{x+1} \right) dx = \ln|x| + \ln|x-1| - \ln|x+1| + C$$