

# Integration by Parts (TRIG)

$$\int \tan^2 \theta \sec \theta \, d\theta = \int \sec^3 \theta - \int \sec \theta \, d\theta$$

$\downarrow$   
 $\sec^2 \theta \sec \theta$

$$\begin{aligned} \text{let } u &= \sec \theta & v &= \tan \theta \\ du &= \sec \theta \tan \theta \, d\theta & dv &= \sec^2 \theta \end{aligned}$$

$$\begin{aligned} uv - \int v \, du \\ \sec \theta \tan \theta - \int \tan \theta \cdot \sec \theta \tan \theta \, d\theta \end{aligned}$$

$$\int \tan^2 \theta \sec \theta \, d\theta = \sec \theta \tan \theta - \int \tan^2 \theta \sec \theta \, d\theta - \int \sec \theta \, d\theta$$

$$2 \int \tan^2 \theta \sec \theta \, d\theta = \sec \theta \tan \theta - \int \sec \theta \, d\theta$$

$$\int \tan^2 \theta \sec \theta \, d\theta = \frac{1}{2} \sec \theta \tan \theta - \frac{1}{2} \int \sec \theta \, d\theta$$

$$= \frac{1}{2} \sec \theta \tan \theta - \frac{1}{2} \ln |\sec \theta + \tan \theta| + C.$$