Integration by Parts (TRIG)

$$\int \tan^2\theta \sec\theta \, d\theta = \int \sec^3\theta - \int \sec\theta \, d\theta$$

$$\sec^2\theta \sec\theta$$

let $u = \sec\theta$ $\quad v = \tan\theta$

$du = \sec\theta\tan\theta \, d\theta$ $\quad dv = \sec^2\theta$

$$uv - \int v \, du$$

$$\sec\theta\tan\theta - \int \tan\theta \cdot \sec\theta \tan\theta \, d\theta$$

$$\int \tan^2\theta \sec\theta \, d\theta = \sec\theta\tan\theta - \int \tan^2\theta \sec\theta \, d\theta - \int \sec\theta \, d\theta$$

$$2\int \tan^2\theta \sec\theta \, d\theta = \sec\theta\tan\theta - \int \sec\theta \, d\theta$$

$$\int \tan^2\theta \sec\theta \, d\theta = \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\int \sec\theta \, d\theta$$

$$= \boxed{\frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln|\sec\theta + \tan\theta| + C.}$$