Ch 6

Work Problems

(2) How much work is done when a hoist lifts a 200-kg rock to a height of 3m?

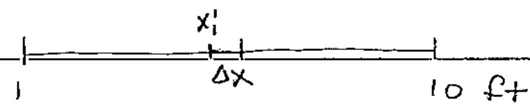
$$F = ma = m \cdot g = 200(9.8) \text{ Newtons}$$

$$W = F \cdot d$$

$$= 200(9.8)(3) \text{ Newton-meters}$$

$$= \boxed{5880 \text{ J}}$$

(3) A variable force of $5x^{-2}$ pounds moves an object along a straight line when it is x feet from the origin. Calculate the work done in moving the object from $x=1$ ft to $x=10$ ft.



$$\Delta W = f(x)\,\Delta x$$

$$= 5x^{-2}\,\Delta x$$

$$W = \int_1^{10} f(x)\,dx$$

$$= \int_1^{10} 5x^{-2}\,dx = \left.\frac{-5}{x}\right|_1^{10} = \frac{-5}{10} - -5$$

$$= -\frac{1}{2} + 5$$

$$= \boxed{4.5 \text{ ft-lbs.}}$$

Work Problems

(10) If the work required to stretch a spring 1 ft beyond its natural length is 12 ft-lb, how much work is needed to stretch it 9 in. beyond its natural length?

$$W = \int_0^1 F(x)\,dx \qquad \text{where } F(x) = kx$$

$$12 = \int_0^1 kx\,dx = \frac{kx^2}{2}\Big|_0^1 = \frac{k}{2} - 0$$

$$12 = \frac{k}{2} \Rightarrow k = 24$$

$$F(x) = 24x$$

$$9 \text{ inches} \cdot \frac{1 \text{ ft}}{12 \text{ inches}} = \frac{9}{12} \cancel{\text{ft}} = \frac{3}{4} \cancel{\text{ft}}$$

$$W = \int_0^{3/4} 24x\,dx = \frac{24x^2}{2}\Big|_0^{3/4} = 12x^2\Big|_0^{3/4}$$

$$= \overset{3}{\cancel{12}} \cdot \frac{9}{\underset{4}{\cancel{16}}} - 0$$

$$= \frac{27}{4} \text{ ft-lbs.}$$

$$\approx \boxed{6.75 \text{ ft-lbs}}$$

Work Problems

(12) If 6 J of work is needed to stretch a spring from 10cm to 12cm and another 10J is needed to stretch it from 12 cm to 14cm, what is the natural length of the spring.

let n = natural length of spring

$$6J = \int_{10-n}^{12-n} kx\,dx = \frac{kx^2}{2}\Big|_{10-n}^{12-n} = \frac{k(12-n)^2}{2} - \frac{k(10-n)^2}{2}$$

$$= \frac{k(144-24n+n^2) - k(100-20n+n^2)}{2}$$

$$= \frac{44k-4n}{2}$$

$$\boxed{6 = 22k-2n}$$

$$10J = \int_{12-n}^{14-n} kx\,dx = \frac{kx^2}{2}\Big|_{12-n}^{14-n} = \frac{k(14-n)^2}{2} - \frac{k(12-n)^2}{2}$$

$$= \frac{k(196-28n+n^2) - k(144-24n+n^2)}{2}$$

$$10 = \frac{52k-4n}{2}$$

$$10 = 26k-2n$$

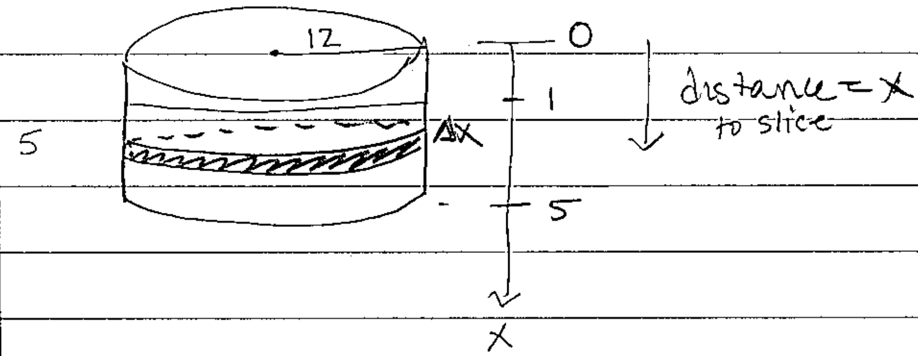
$14 \times 14 = 56 + 140 = 196$; $196 - 144 = 52$

$$22k-2n = 6$$
$$-26k+2n = -10$$
$$-4k = -4$$
$$k = 1 \rightarrow 22k-2n = 6$$
$$22-2n = 6$$
$$16 = 2n$$
$$\boxed{8 = n}$$

Natural length of spring is 8cm.

Work problems

(20) A circular swimming has a diameter of 24 ft, the sides are 5 ft high, and the depth of the water is 4 ft. How much work is required to pump all of the water out over the side? (H_2O weighs 62.5 lb/ft^3)



$$\text{Force} = \text{lbs} = \underset{\text{slice}}{\text{Volume}} \cdot 62.5 \frac{\text{lb}}{\text{ft}^3}$$

$$= \pi (12)^2 \Delta x \cancel{\text{ft}^3} \cdot 62.5 \frac{\text{lbs}}{\cancel{\text{ft}^3}}$$

$$\Delta \text{Force} = (144)(62.5)\pi \Delta x \text{ lbs}$$

$$\Delta \text{Work} = \Delta\text{Force} \cdot \text{distance}$$

$$= (144)(62.5)\pi \cdot x \cdot \Delta x$$

$$\text{Total Work} = \int_1^5 (144)(62.5)\pi x \, dx$$

$$(144)(62.5)\pi \frac{x^2}{2} \Big|_1^5 = (144)(62.5)\pi \cdot \left(\frac{25}{2} - \frac{1}{2}\right)$$

$$= (144)(62.5)(12)\pi \text{ ft lbs}$$

$$\boxed{108{,}000\pi \text{ ft-lbs}}$$