Ch 8 Homework #3

Find the arc length of the curve

(3) $y = \int_0^x \sqrt{4\sin^2 t - 1}\, dt$ from $0 \le x \le \frac{\pi}{2}$

Build the ds

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$y = \int_0^x \sqrt{4\sin^2 t - 1}\, dt$$

$$\frac{dy}{dx} = \frac{d}{dx}\int_0^x \sqrt{4\sin^2 t - 1}\, dt = \sqrt{4\sin^2 x - 1}$$

By FTC 1.

$$\left(\frac{dy}{dx}\right)^2 = 4\sin^2 x - 1$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + 4\sin^2 x - 1 = 4\sin^2 x$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= 2\sin x\, dx$$

$$\text{Length} = \int_0^{\pi/2} 2\sin x\, dx = -2\cos x \Big|_0^{\pi/2}$$

$$= -2\cos\frac{\pi}{2} + 2\cos 0$$

$$= 2(1) = \boxed{2}$$