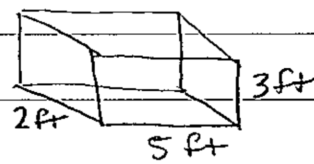
8.3 Hydrostatic pressure, force problems

(1) An aquarium 5ft long, 2 ft wide, and 3ft deep is full of water. Find (a) the hydrostatic pressure on the bottom of the aquarium, (b) the hydrostatic force on the bottom and (c) the hydrostatic force on one end of the aquarium.



(a) hydrostatic Pressure on the bottom of the tank.

$P = \delta d$ where δ = density of water = $62.5\ lb/ft^3$

d = depth of aquarium = 3 ft.

$= (62.5)\frac{lb}{ft^3} \cdot 3\,ft = \boxed{187.5\ lb/ft^2}$

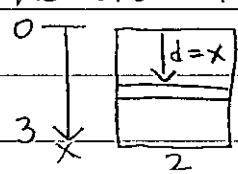
(b) hydrostatic force on the bottom

$F = P \cdot A$

$= (187.5)\frac{lb}{ft^2} \cdot (5ft)(2ft) = \boxed{1{,}875\ lbs}$

(c) hydrostatic force on one end of the aquarium

The end of the tank has dimensions 2ft x 3ft



$\Delta F = \Delta(PA)$

$\Delta P = \delta \cdot d = 62.5 \cdot x$

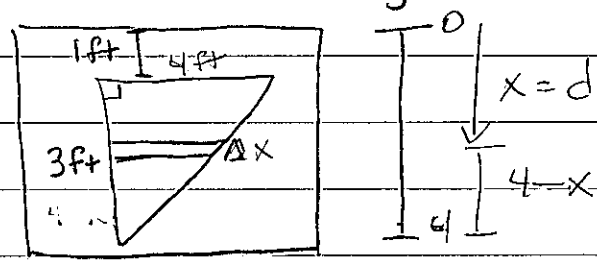
$\Delta A = 2\Delta x$

$\Delta(PA) = 2(62.5)x\Delta x = 125x\Delta x$

$\text{Pressure} = \int_0^3 125x\,dx = \frac{125x^2}{2}\Big|_0^3 = \frac{(125)(9)}{2} - 0 = \boxed{562.5\ lb}$

Hydrostatic Force

(4) A vertical plate is submerged in water as shown. Find the hydrostatic force against the side of the plate.



$\Delta F = \Delta(PA)$

$\Delta P = \delta d = 62.5x$

$\Delta A = h \cdot \Delta x$

$= \frac{4}{3}(4-x)\Delta x$

By similar triangles

$\frac{4}{3} = \frac{h}{4-x}$ $\quad h = \frac{4}{3}(4-x)$

$\Delta(PA) = \frac{4}{3}(4-x)\,x\,(62.5)\,\Delta x$

$$F = \int_1^4 \frac{4}{3}(4-x)\,x\,(62.5)\,dx = \frac{(62.5)(4)}{3}\int (4x - x^2)\,dx$$

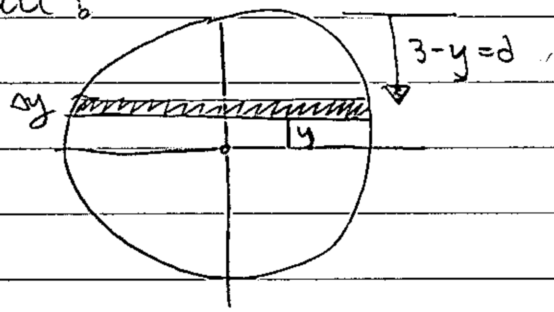
$$= \frac{(62.5)(4)}{3} \cdot \left(\frac{4x^2}{2} - \frac{x^3}{3}\right)\Big|_1^4 = \frac{(62.5)(4)}{3}\left(32 - \frac{64}{3} - 2 + \frac{1}{3}\right)$$

$$= \frac{(62.5)(4)(9)}{3}$$

$$= \boxed{750 \text{ lbs}}$$

Hydrostatic Force

(12) A milk truck carries milk with density 64.6 lb/ft^3 in a horizontal cylindrical tank with diameter 6 ft. (a) Find the force exerted by the milk on one end of the tank when the tank is full. (b) What if the tank is half full?



$x^2+y^2=9$

$x = \sqrt{9-y^2}$

$\Delta F = \Delta(PA)$

$\Delta P = \delta \cdot d = (64.6)(3-y)$

$\Delta A = L \cdot W = 2\sqrt{9-y^2}\,\Delta y$

$\Delta F = \Delta(PA) = (64.6)(3-y)\,2\sqrt{9-y^2}\,\Delta y$

$$F = \int_{-3}^{3} 129.2\,(3-y)\sqrt{9-y^2}\,dy$$

$$= (129.2)(3)\int_{-3}^{3}\sqrt{9-y^2}\,dy - 129.2\int_{-3}^{3} y\sqrt{9-y^2}\,dy$$

↑ odd function

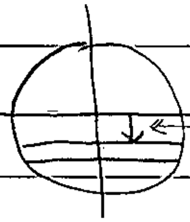
$$= (129.2)(3)\int_{-3}^{3}\sqrt{9-y^2}\,dy - 0$$

↑ semicircle radius 3 (area)

$$= (129.2)(3)\cdot\frac{1}{2}\pi(3)^2 = 1744.2\pi \approx 5480 \text{ lb.}$$

(b) If the tank is half full, the surface of the milk is $y=0$, so $\Delta P = \delta d = (64.6)(-y) = -64.6y$

$\Delta A = L \cdot W = 2\sqrt{9-y^2} \cdot \Delta y$



$\Delta F = \Delta P = -64.6y\left(2\sqrt{9-y^2}\right)\Delta y$

$= -129.2y\sqrt{9-y^2}\,\Delta y$

$$\text{Force} = \int_{-3}^{0} -129.2y\sqrt{9-y^2}\,dy$$

$u = 9-y^2$ $\quad y=-3$ $u=0$

$du = -2y\,dy$ $\quad y=0$ $u=9$

$$= \int_0^9 64.6\,u^{1/2}\,du$$

$$= 64.6 \cdot u^{3/2} \cdot \frac{2}{3}\Big|_0^9 = \frac{129.2}{3}(27) = \boxed{1162.8 \text{ lbs}}$$

about 20% of full tank force