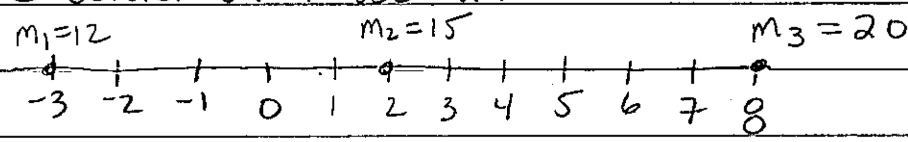
Ch8.3

Moment and Center of mass.

(22) Point masses m_i are located on the x-axis as shown. Find the moment M of the system about the origin and the center of mass $\bar{x}$.



$$M_0 = m_1\cdot(-3) + m_2(2) + m_3(8)$$
$$= 12(-3) + 15(2) + 20(8)$$
$$= -36 + 30 + 160$$

$\boxed{M_0 = 154}$ moment about origin

mass $m = 12 + 15 + 20 = 47$

center of mass $\boxed{\bar{x} = \dfrac{M_0}{m} = \dfrac{154}{47}}$

(24) The masses m_i are located at the points P_i. Find the moments M_x and M_y and the center of mass of the system.

$m_1 = 5$ $m_2 = 4$ $m_3 = 3$ $m_4 = 6$

$P_1(-4,2)$, $P_2(0,5)$ $P_3(3,2)$ $P_4(1,-2)$

$$M_x = \sum_{i=1}^{4} m_i y_i = (5)(2) + (4)(5) + (3)(2) + 6(-2)$$
$$= 10 + 20 + 6 - 12$$
$$= 24$$

$$M_y = \sum_{i=1}^{4} m_i x_i = (5)(-4) + 4(0) + 3(3) + 6(1)$$
$$= -20 + 0 + 9 + 6$$
$$= -5$$

Total mass $m = \sum m_i = 5+4+3+6 = 18$

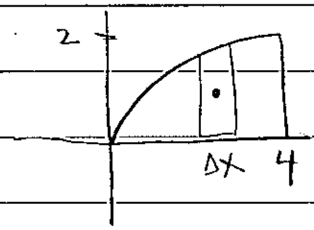
$$\bar{x} = \frac{M_y}{m} = \frac{-5}{18}$$

$$\bar{y} = \frac{M_x}{m} = \frac{24}{18} = \frac{4}{3}$$

$$\boxed{(\bar{x}, \bar{y}) = \left(\frac{-5}{18}, \frac{4}{3}\right)}$$

(26) Sketch the region bounded by the curves, and visually estimate the location of the centroid. Then find the exact coordinates of the centroid.

$y = \sqrt{x}$, $y = 0$, $x = 4$



$$\int_a^b f(x)\,dx = \int_0^4 x^{1/2}\,dx = \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{2}{3}(8) - 0 = \frac{16}{3}$$

$$\int_a^b x f(x)\,dx = \int_0^4 x^{3/2}\,dx = \frac{2}{5}x^{5/2}\Big|_0^4 = \frac{2}{5}(32) = \frac{64}{5}$$

$$\int_a^b \frac{1}{2}(f(x))^2\,dx = \frac{1}{2}\int_0^4 x\,dx = \frac{1}{2}\,\frac{x^2}{2} = \frac{x^2}{4}\Big|_0^4 = 4 - 0 = 4$$

$$\bar{x} = \frac{M_y}{m} = \frac{\rho\int_a^b x f(x)\,dx}{\rho\int_a^b f(x)\,dx} = \frac{\frac{64}{5}}{\frac{16}{3}} = \frac{64}{5}\cdot\frac{3}{16} = \frac{12}{5}$$

$$\bar{y} = \frac{M_x}{m} = \frac{\rho\int_a^b \frac{1}{2}(f(x))^2\,dx}{\rho\int_a^b f(x)\,dx} = \frac{4}{\frac{16}{3}} = \frac{12}{16} = \frac{3}{4}$$

$$\boxed{(\bar{x}, \bar{y}) = \left(\frac{12}{5}, \frac{3}{4}\right)}$$