6.4 (2) Find the length of the indicated curve.

$y = \frac{2}{3}(x^2+1)^{3/2}$ between $x=1$ and $x=2$

$$L = \int_1^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$y = \frac{2}{3}(x^2+1)^{3/2}$$

$$y' = \frac{2}{3}\cdot\frac{3}{2}(x^2+1)^{1/2}\cdot 2x$$

$$y' = 2x\sqrt{x^2+1}$$

$$= \int_1^2 \sqrt{1+4x^2(x^2+1)}\,dx$$

$$= \int_1^2 \sqrt{1+4x^4+4x^2}\,dx$$

$$= \int_1^2 \sqrt{4x^4+4x^2+1}\,dx$$

$$= \int_1^2 \sqrt{(2x^2+1)^2}\,dx$$

$$= \int_1^2 2x^2+1\,dx = \left.\frac{2x^3}{3}+x\right|_1^2$$

$$= \left(\frac{16}{3}+2\right)-\left(\frac{2}{3}+1\right)$$

$$= \frac{14}{3}+\frac{3}{3} = \boxed{\frac{17}{3}}$$

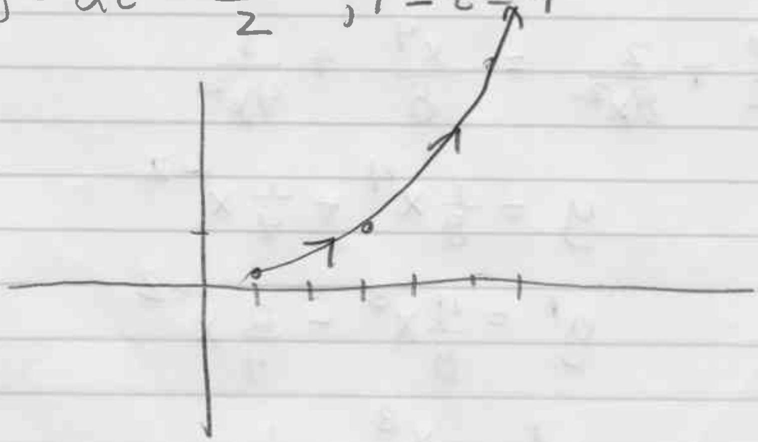
$127\frac{1}{2}$
$\times 2$
255

6.4 (10) Sketch the graph of the given parametric equation and find its length

$x = 3t^2 + 2$

$y = 2t^3 - \frac{1}{2}$, $1 \le t \le 4$

t	x	y
1	5	3/2
2	14	31/2
3	29	107/2
4	48	255/2



$x = 3t^2 + 2$ $\qquad$ $y = 2t^3 - \frac{1}{2}$

$\frac{dx}{dt} = 6t$ $\qquad$ $\frac{dy}{dt} = 6t^2$

$$L = \int_1^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_1^4 \sqrt{(6t)^2 + (6t^2)^2}\, dt = \int_1^4 \sqrt{36t^2 + 36t^4}\, dt$$

$$= \int_1^4 \sqrt{36t^2(1+t^2)}\, dt = \int_1^4 6t\sqrt{1+t^2}\, dt$$

$$= 3\int_1^4 2t\sqrt{1+t^2}\, dt = 3\int_2^{17} u^{1/2}\, du = \cancel{3} \cdot u^{3/2} \cdot \frac{2}{\cancel{3}} \Big|_2^{17}$$

let $u = 1+t^2$ $\quad du = 2t\,dt$

$t=1$ $u=2$, $t=4$ $u=17$

$$= 2\sqrt{17^3} - 2\sqrt{2^3}$$

$$L = \boxed{2(17\sqrt{17} - 2\sqrt{2})}$$

6.4 | 28 Find the area of the surface generated by revolving the given curve about the x-axis.

$$y = \frac{(x^6+2)}{(8x^2)} \qquad 1 \le x \le 3$$

$$y = \frac{x^6}{8x^2} + \frac{2}{8x^2} = \frac{x^4}{8} + \frac{1}{4x^2}$$

$$y = \frac{1}{8}x^4 + \frac{1}{4}x^{-2}$$

$$y' = \frac{4}{8}x^3 - \frac{2}{4}x^{-3}$$

$$y' = \frac{x^3}{2} - \frac{1}{2x^3}$$

Surface Area $= 2\pi \int_1^3 y \, ds$ where $y = \frac{x^6+2}{8x^2}$

$$ds = \sqrt{1+(y')^2}\, dx$$

$$= 2\pi \int_1^3 \left(\frac{x^4}{8} + \frac{1}{4x^2}\right)\sqrt{1+\left(\frac{x^3}{2} - \frac{1}{2x^3}\right)^2}\, dx$$

$$= 2\pi \int_1^3 \left(\frac{x^4}{8} + \frac{1}{4x^2}\right)\sqrt{1+\frac{x^6}{4} - 2\cdot\frac{\cancel{x^3}}{\cancel{4}_2\cancel{x^3}} + \frac{1}{4x^6}}\, dx$$

$$= 2\pi \int_1^3 \left(\frac{x^4}{8} + \frac{1}{4x^2}\right)\sqrt{\frac{x^6}{4} + \frac{1}{2} + \frac{1}{4x^6}}\, dx$$

$$= 2\pi \int_1^3 \left(\frac{x^4}{8} + \frac{1}{4x^2}\right)\sqrt{\left(\frac{x^3}{2} + \frac{1}{2x^3}\right)^2}\, dx$$

6.4 28 continued...

$$\text{Surface Area} = 2\pi \int_1^3 \left(\frac{x^4}{8} + \frac{1}{4x^2}\right)\left(\frac{x^3}{2} + \frac{1}{2x^3}\right) dx$$

$$= 2\pi \int_1^3 \left(\frac{x^7}{16} + \frac{x}{16} + \frac{x}{8} + \frac{1}{8x^5}\right) dx$$

$$= 2\pi \int_1^3 \frac{x^7}{16} + \frac{3x}{16} + \frac{1}{8x^5}\Big) dx$$

$$= 2\pi \int_1^3 \left(\frac{1}{16}x^7 + \frac{3}{16}x + \frac{1}{8}x^{-5}\right) dx$$

$$= 2\pi \left(\frac{x^8}{128} + \frac{3x^2}{32} - \frac{1}{32x^4}\right)\Bigg|_1^3$$

$$= 2\pi \left[\left(\frac{6561}{128} + \frac{27}{32} - \frac{1}{2592}\right) - \left(\frac{1}{128} + \frac{3}{32} - \frac{1}{32}\right)\right]$$

$$= \boxed{\frac{8429\pi}{81}}$$