Math 3B - Fall 10

Sec 4.9 Notes. 8/26/10

Exercises

(1) Show that $y=\sqrt{1-x^2}$ is a solution to the differential equation

$$\frac{dy}{dx}+\frac{x}{y}=0$$

$$y=\sqrt{1-x^2}=(1-x^2)^{1/2}$$

$$\frac{dy}{dx}=\frac{1}{2}(1-x^2)^{-1/2}\cdot -2x \qquad \text{Chain Rule!}$$

$$\frac{dy}{dx}=\frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}=\frac{-x}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx}+\frac{x}{y}=0$$ ← Equation, Plug-in $\frac{dy}{dx}$ and y and verify true!

$$\left(\frac{-x}{\sqrt{1-x^2}}\right)+\frac{x}{\sqrt{1-x^2}}=\frac{-x+x}{\sqrt{1-x^2}}=\frac{0}{\sqrt{1-x^2}}=0 \checkmark$$

(6) Find the general solution (involving constant C) Then find the particular equation that satisfies the indicated condition.

$$\frac{dy}{dx}=x^{-3}+2 \qquad y=3 \text{ at } x=1$$

$$dy=(x^{-3}+2)\,dx$$

$$\int dy=\int (x^{-3}+2)\,dx$$

$$\boxed{\begin{aligned} y &= \frac{x^{-2}}{-2}+2x+C \\ y &= \frac{-1}{2x^2}+2x+C \end{aligned}}$$

$$\boxed{y=\frac{-1}{2x^2}+2x+\frac{3}{2}}$$
particular solution

↓ ORIGINAL CONDITIONS

$y=3$ when $x=1$

$$3=\frac{-1}{2(1)^2}+2(1)+C$$

$$3=-\frac{1}{2}+2+C$$

$$3=1.5+C$$

$$1.5=3/2=C$$

Find the general solution & particular solution

(8) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y = 4$ at $x = 1$

$= \frac{\sqrt{x}}{\sqrt{y}}$

$\sqrt{y}\,dy = \sqrt{x}\,dx$

$\int y^{1/2}dy = \int x^{1/2}dx$

$\frac{2}{3}y^{3/2} + C_1 = \frac{2}{3}x^{3/2} + C_2$

$\frac{2}{3}y^{3/2} = \frac{2}{3}x^{3/2} \underbrace{+C_2 - C_1}$

$\frac{2}{3}y^{3/2} = \frac{2}{3}x^{3/2} + C_3$

$y^{3/2} = x^{3/2} + \frac{C_3}{\frac{2}{3}}$

$y^{3/2} = x^{3/2} + C_4$

$\boxed{y = \left(x^{3/2} + C\right)^{2/3} \quad \text{general solution}}$

$4 = \left(1^{3/2} + c\right)^{2/3}$

$4^3 = (1 + c)^2$

$64 = (1 + C)^2$

$8 = 1 + C$

$7 = C$

$\boxed{y = \left(x^{3/2} + 7\right)^{2/3} \quad \text{particular solution.}}$

4.9/#12 Find the general solution and the particular solution.

$$\frac{du}{dt} = u^3(t^3 - t) \qquad u=4 \text{ at } t=0$$

$$\frac{du}{u^3} = (t^3 - t)dt$$

$$\int u^{-3}du = \int (t^3 - t)\,dt$$

$$\frac{u^{-2}}{-2} + C_1 = \frac{t^4}{4} - \frac{t^2}{2} + C_2$$

$$-\frac{1}{2u^2} = \frac{t^4}{4} - \frac{t^2}{2} + C_3$$

$$\frac{1}{u^2} = t^2 - \frac{t^4}{2} + C$$

$$\boxed{u = \left(t^2 - \frac{t^4}{2} + c\right)^{-1/2} \quad \text{general solution}}$$

$$4 = \left(0^2 - \frac{0^4}{2} + c\right)^{-1/2}$$

$$4 = c^{-1/2}$$

$$4 = \frac{1}{\sqrt{c}}$$

$$16 = \frac{1}{C}$$

$$C = \frac{1}{16}$$

$$\boxed{u = \left(t^2 - \frac{t^4}{2} + \frac{1}{16}\right)^{-1/2} \quad \text{particular solution}}$$

(16) Find the xy-equation of the curve through (1,2) whose slope is 3 times the square of its y-coord.

$$\frac{dy}{dx} = 3 \cdot y^2 \quad \text{at } (1,2)$$

$x=1, y=2$

$$y^{-2}\,dy = 3\,dx$$

$$\int y^{-2}\,dy = \int 3\,dx$$

$$\frac{y^{-1}}{-1} + C_1 = 3x + C_2$$

$$\frac{-1}{y} + C_1 = 3x + C_2$$

$$\frac{-1}{y} = 3x + C$$

$$y = \frac{-1}{3x+C}$$

at $x=1, y=2$ (point $(1,2)$).

$$2 = \frac{-1}{3(1)+C}$$

$$2 = \frac{-1}{3+C}$$

$$6 + 2C = -1$$

$$2C = -7$$

$$C = -7/2$$

solution

$$\boxed{y = \frac{2}{7-6x}}$$

$$y = \frac{-1}{3x - 7/2} = \frac{-1}{\frac{6x-7}{2}} = \frac{-2}{6x-7} = \frac{2}{7-6x}$$

4.10

(12) The mass of a tumor grows at a rate proportional to its size. The first measurement of its mass was 4.0 grams. Four months later its mass was 6.76 gms. How large was the tumor 6 months before the first measurement? If the instrument can detect tumors of mass 1 gram or greater, would the tumor have been detected at that time?

$t = 0 \quad m = \underline{4}$

$t = 4 \quad m = 6.76$

$t = -6 \quad m = \underline{\qquad}$

Find m_0
Find k

$$\frac{dm}{dt} = km \longrightarrow m = m_0 e^{kt}$$

exponential growth model

at $t = 0 \quad m = 4$

$$m = m_0 e^{kt}$$
$$4 = m_0 e^0$$
$$\boxed{4 = m_0}$$
$$m = 4e^{kt}$$

$t = 4 \quad m = 6.76$

$$6.76 = 4e^{4k}$$
$$\frac{6.76}{4} = e^{4k}$$
$$\ln\left(\frac{6.76}{4}\right) = 4k$$
$$.1312 \approx \frac{.5247}{4} \approx \frac{\ln\left(\frac{6.76}{4}\right)}{4} = k \qquad \boxed{k = .1312}$$

Plug in m_0 & k

get equation $\longrightarrow m = 4e^{.1312t}$ calculate at $t = -6$.

$$m = 4e^{(.1312)(-6)} \approx 1.82 \text{ grams}$$

yes detectable!