7.1/ 21 $\int \frac{6e^x}{\sqrt{1-e^{2x}}}\,dx$

use $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right)+C$

let $u = e^x$
$du = e^x\,dx$

$a = 1$
$a^2 = 1^2$
$u^2 = (e^x)^2$
$= e^{2x}$

$= 6\int \frac{du}{\sqrt{a^2-u^2}} = 6\sin^{-1}\left(\frac{u}{a}\right)+C$

$= 6\sin^{-1}\left(\frac{e^x}{1}\right)+C$

$= \boxed{6\sin^{-1}(e^x)+C}$

28 $\int \frac{\sin(4t-1)}{1-\sin^2(4t-1)}\,dt = \int \frac{\sin(4t-1)}{\cos^2(4t-1)}\,dt$

$= \int \sec(4t-1)\tan(4t-1)\,dt$

let $u = 4t-1$
$du = 4\,dt$

$= \frac{1}{4}\int \sec(u)\tan(u)\,du$

$= \frac{1}{4}\sec(u) + C$

$= \boxed{\frac{1}{4}\sec(4t-1) + C}$

$$\int u\,dv = uv - \int v\,du$$

7.2

(2) $\int xe^{3x}\,dx$

let $u = x$ $\quad v = \frac{1}{3}e^{3x}$

$du = dx \quad dv = e^{3x}\,dx$

$$\int xe^{3x}\,dx = u\cdot v - \int v\cdot du$$

$$= (x)\cdot\frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x}\,dx$$

$u' = 3x$

$du' = 3dx$

$$= \frac{xe^{3x}}{3} - \frac{1}{9}\int 3e^{3x}\,dx$$

$$= \boxed{\frac{xe^{3x}}{3} - \frac{1}{9}e^{3x} + C}$$

(8) $\int \underbrace{(x-\pi)}_{u}\underbrace{\sin x\,dx}_{dv}$

let $u = x-\pi \quad V = -\cos x$

$du = dx \quad dv = \sin x\,dx$

$$= u\cdot v - \int v\cdot du$$

$$= (x-\pi)(-\cos x) - \int(-\cos x)\,dx$$

$$= -(x-\pi)\cos x + \int \cos x\,dx$$

$$= \boxed{-(x-\pi)\cos x + \sin x + C}$$

OR $(\pi - x)\cos x + \sin x + C$

distribute the negative!

10 $\int t \sqrt[3]{2t+7}$

let $u = t$ $\quad$ $v = \frac{3}{8}(2t+7)^{4/3}$

$du = dt$ $\quad$ $dv = (2t+7)^{1/3}dt$

$= u \cdot v - \int v \cdot du$

$= (t)\left(\frac{3}{8}(2t+7)^{4/3}\right) - \int \frac{3}{8}(2t+7)^{4/3}\,dt$

$= \frac{3t}{8}(2t+7)^{4/3} - \frac{3}{8}\int (2t+7)^{4/3}\,dt$

let $u = 2t+7$

$du = 2\,dt$

$= \frac{3t}{8}(2t+7)^{4/3} - \frac{3}{8} \cdot \frac{1}{2}\int 2(2t+7)^{4/3}\,dt$

$= \frac{3t}{8}(2t+7)^{4/3} - \frac{3}{16}\int u^{4/3}\,du$

$= \frac{3t}{8}(2t+7)^{4/3} - \frac{3}{16} \cdot \frac{3}{7}u^{7/3} + C$

$= \boxed{\frac{3t}{8}(2t+7)^{4/3} - \frac{9}{112}(2t+7)^{7/3} + C}$

$\int dv = \frac{1}{2}\int 2\sqrt[3]{2t+7}\,dt$

$u' = 2t+7$

$du' = 2\,dt$

$= \frac{1}{2}\int (u')^{1/3}\,dt$

$= \frac{1}{2} \cdot \frac{(u')^{4/3}}{4/3}$

$= \underbrace{\frac{1}{2} \cdot \frac{3}{4}(2t+7)^{4/3}}_{v}$

7.3 Trigonometric Integrals

① $\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx$

$= \int \frac{1}{2} - \frac{\cos 2x}{2}\,dx$

$= \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x\,dx$

let $u = 2x$

$du = 2dx$

$= \frac{1}{2}\int dx - \frac{1}{4}\int \cos 2x \cdot 2dx$

$= \frac{1}{2}\int dx - \frac{1}{4}\int \cos u\,du$

$= \boxed{\frac{1}{2}x - \frac{1}{4}\sin 2x + C}$

7.3

5 $\int_0^{\pi/2} \cos^5\theta \, d\theta = \int_0^{\pi/2} \cos\theta \cos^4\theta \, d\theta$

$= \int_0^{\pi/2} \cos\theta \, (\cos^2\theta)^2 \, d\theta$

$= \int_0^{\pi/2} \cos\theta \, (1-\sin^2\theta)^2 \, d\theta$

$= \int_0^{\pi/2} \cos\theta \left(1^2 - 2\sin^2\theta + \sin^4\theta\right) d\theta$

let $u = \sin\theta$

$du = \cos\theta \, d\theta$

$= \int_0^{\pi/2} \left(1 - 2(\sin\theta)^2 + (\sin\theta)^4\right) \cos\theta \, d\theta$

$= \int_0^{\pi/2} (1 - 2u^2 + u^4) \, du$

$= u - \frac{2u^3}{3} + \frac{u^5}{5}$

$= \sin\theta - \frac{2(\sin\theta)^3}{3} + \frac{(\sin\theta)^5}{5} \Big|_0^{\pi/2}$

$= \sin\frac{\pi}{2} - \frac{2[\sin(\pi/2)]^3}{3} + \frac{(\sin \pi/2)^5}{5} - 0$ ($\sin 0 = 0$

$= 1 - \frac{2}{3}(1)^3 + \frac{1^5}{5}$

$= \frac{15}{15} - \frac{10}{15} + \frac{3}{15} = \boxed{\frac{8}{15}}$

7.3

9) $\int \cos^3 3\theta \sin^{-2} 3\theta \, d\theta$

$= \int \cos 3\theta \cos^2 3\theta \sin^{-2} 3\theta \, d\theta$

$= \int \cos 3\theta \, (1-\sin^2 3\theta) \sin^{-2} 3\theta \, d\theta$

$= \int \cos 3\theta \, (\sin^{-2} 3\theta - \sin^0 3\theta) \, d\theta$

$= \int \cos 3\theta \, (\sin^{-2} 3\theta - 1) \, d\theta$

let $u = \sin 3\theta$

$du = \cos 3\theta \, (3) \, d\theta$

$= 3 \cos 3\theta \, d\theta$

$= \frac{1}{3} \int ((\sin 3\theta)^{-2} - 1) \, 3 \cos 3\theta \, d\theta$

$= \frac{1}{3} \int (u^{-2} - 1) \, du$

$= \frac{1}{3} \left(\frac{u^{-1}}{-1} - u \right) + C$

$= \frac{1}{3} \left(-\frac{1}{u} - u \right) + C$

$= \frac{1}{3} \left(-\frac{1}{\sin 3\theta} - \sin 3\theta \right) + C$

$= \boxed{\frac{-\csc 3\theta}{3} - \frac{\sin 3\theta}{3} + C}$

(13) $\int \sin 4y \cos 5y \, dy$

Use Identity:

$$\sin mx \cos nx = \frac{1}{2}\left[\sin(m+n)x + \sin(m-n)x\right]$$

So

$$\sin 4y \cos 5y = \frac{1}{2}\left[\sin(4+5)y + \sin(4-5)y\right]$$

$$= \frac{1}{2}\left[\sin 9y + \sin(-y)\right]$$

$$= \frac{1}{2}\left[\sin 9y - \sin y\right]$$

So

$$\int \sin 4y \cos 5y \, dy = \int \frac{1}{2}(\sin 9y - \sin y)\,dy$$

$$= \frac{1}{2}\int \sin 9y \, dy - \frac{1}{2}\int \sin y \, dy$$

let $u = 9y$

$du = 9\,dy$

$$= \frac{1}{2}\cdot\frac{1}{9}\int \sin 9y \cdot 9\,dy \quad - \frac{1}{2}\int \sin y\,dy$$

$$= \frac{1}{18}\int \sin u \, du \quad - \frac{1}{2}\int \sin y \, dy$$

$$= \frac{1}{18} - \cos u \quad - \frac{1}{2}(-\cos y) + C$$

$$= \boxed{\frac{-\cos 9y}{18} + \frac{\cos y}{2} + C}$$

7.3

16 $\int \sin 3t \sin t \, dt$

Use product identity:

$$\sin mx \sin nx = -\frac{1}{2}\left[\cos(m+n)x - \cos(m-n)x\right]$$

So

$$\sin 3t \sin t = -\frac{1}{2}\left[\cos(3+1)t - \cos(3-1)t\right]$$

$$= -\frac{1}{2}\left[\cos 4t - \cos 2t\right]$$

Therefore

$$\int \sin 3t \sin t \, dt = \int -\frac{1}{2}\left[\cos 4t - \cos 2t\right] dt$$

$$= -\frac{1}{2}\int \cos 4t \, dt + \frac{1}{2}\int \cos 2t \, dt$$

let $u = 4t$ let $v = 2t$

$du = 4dt$ $dv = 2dt$

$$= -\frac{1}{2}\cdot\frac{1}{4}\int \cos 4t \cdot 4dt + \frac{1}{2}\cdot\frac{1}{2}\int \cos 2t \cdot 2dt$$

$$= -\frac{1}{8}\int \cos u \, du + \frac{1}{4}\int \cos v \, dv$$

$$= -\frac{1}{8}\sin u + \frac{1}{4}\sin v + C$$

$$= \boxed{\frac{-\sin 4t}{8} + \frac{\sin 2t}{4} + C}$$