8.3

(8) Evaluate the improper integral or show that it diverges

$$\int_9^{\infty} \frac{x\,dx}{\sqrt{1+x^2}}$$

let $u = 1+x^2$

$du = 2x\,dx$

$$= \frac{1}{2}\int u^{-1/2}du = \frac{1}{2}\cdot\frac{u^{+1/2}}{\frac{1}{2}} = u^{1/2}$$

$$= (1+x^2)^{1/2}$$

$$= \sqrt{1+x^2}\,\Big|_9^{\infty}$$

$$= \lim_{b\to\infty} F(b) - F(9)$$

$$= \lim_{b\to\infty} \sqrt{1+b^2} - \sqrt{1+9^2} = \infty - \sqrt{82} = \infty$$

$\boxed{\text{DIVERGES}}$

8.3 Improper Integrals

(11) $\int_e^{\infty} \frac{1}{x \ln x}\, dx$

let $u = \ln x$

$du = \frac{1}{x} dx$

$= \int \frac{du}{u} = \ln u$

$= \ln[\ln x] \Big|_e^{\infty}$

$= \lim_{b \to \infty} F(b) - F(e)$

$= \lim_{b \to \infty} \ln[\ln b] - \ln[\ln e]$

$= \infty - 0 = \infty$

$\boxed{\text{The Integral Diverges}}$

8.3 Improper Integrals.

(14) $\int_1^{\infty} xe^{-x}\,dx$

By parts

let $u = x$ $\quad v = -e^{-x}$

$du = dx$ $\quad dv = e^{-x}dx$

$$\int_1^{\infty} xe^{-x}\,dx = u \cdot v - \int v \cdot du$$

$$= -xe^{-x} - \int -e^{-x}\,dx$$

$$= -xe^{-x} - e^{-x} \Big|_1^{\infty}$$

$$= \lim_{b\to\infty} F(b) - F(1)$$

$$= \lim_{b\to\infty} \left[-be^{-b} - e^{-b}\right] - \left[-1e^{-1} - e^{-1}\right]$$

$$= \lim_{b\to\infty} \left[\frac{-b}{e^b} - \frac{1}{e^b}\right] + \frac{2}{e}$$

By sec 8.1 L.H Rule Applies $\left(\frac{\infty}{\infty}\right)$

$$= \lim_{b\to\infty} \frac{-b}{e^b} + \lim_{b\to\infty}\left(\frac{-1}{e^b} + \frac{2}{e}\right)$$

$$= 0 + 0 + \frac{2}{e}$$

$$= \boxed{\frac{2}{e}}$$