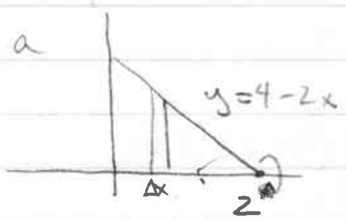
6.2 (4) Find the volume of the solid generated when the indicated region is revolved about the specified axis; slice, approximate, integrate.

a



$\boxed{\text{Find volume about x-axis}}$

$$V = \pi\int_0^2 (4-2x)^2\,dx = \pi\int_0^2 (16-16x+4x^2)\,dx$$

$$= \pi\left(16x - \frac{16x^2}{2} + \frac{4x^3}{3}\right)\Big|_0^2$$

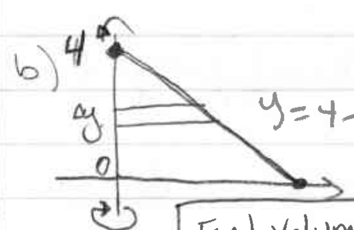
$$= \pi\left(32 - 8(4) + \frac{4}{3}\cdot 8\right)$$

$$= \pi\cdot\frac{32}{3} \approx \boxed{\frac{32\pi}{3}}$$

$y = 0 = 4-2x = 4 = 2x$

$2 = x$

b)



$\boxed{\text{Find volume about y axis}}$

$y + 2x = 4$

$2x = 4 - y$

$x = \frac{4-y}{2} = 0$

$4 - y = 0$

$\boxed{y = 4}$ upper limit of integration

$$V = \pi\int_0^4 \left(\frac{4-y}{2}\right)^2 dy = \pi\int_0^4 \left(2 - \frac{y}{2}\right)^2 dy = \pi\int_0^4 \left(4 - 2y + \frac{y^2}{4}\right) dy$$

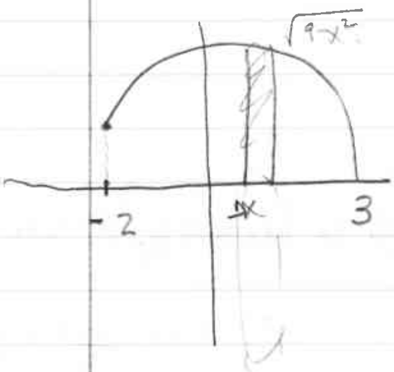
$$= \pi\left(4y - \frac{2y^2}{2} + \frac{y^3}{12}\right)\Big|_0^4$$

$$= 16\pi - 16\pi + \frac{64\pi}{12} - 0$$

$$= \frac{4\cdot 16\pi}{4\cdot 3} = \frac{16\pi}{3}$$

6.2/ 9 $y = \sqrt{9 - x^2}$, $y = 0$ between $x = -2$ and $x = 3$

Find volume generated by revolving $y = \sqrt{9-x^2}$ about x-axis.



$$\text{Volume} = \int_{-2}^{3} \pi \left(\sqrt{9-x^2}\right)^2 dx$$

$$= \pi \int_{-2}^{3} (9 - x^2)\, dx$$

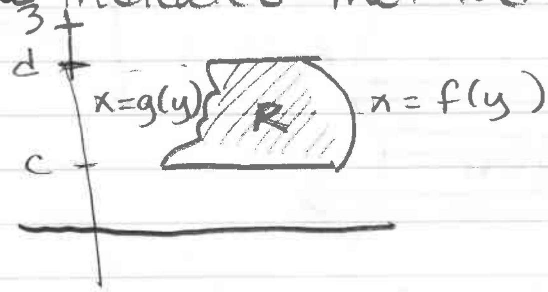
$$= \pi \left(9x - \frac{x^3}{3}\right) \Big|_{-2}^{3}$$

$$= \pi\left(27 - \frac{27}{3}\right) - \pi\left(-18 + \frac{8}{3}\right)$$

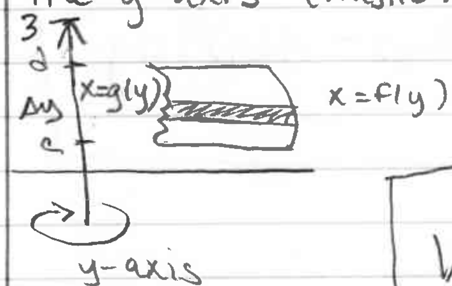
$$= 18\pi + 18\pi - \frac{8\pi}{3}$$

$$= 36\pi - \frac{8\pi}{3} = \boxed{\frac{100\pi}{3}}$$

6.3 (14) A region R is shown. Set up an integral for the volume of the solid obtained when R is revolved about each of the following lines. Use the indicated method.



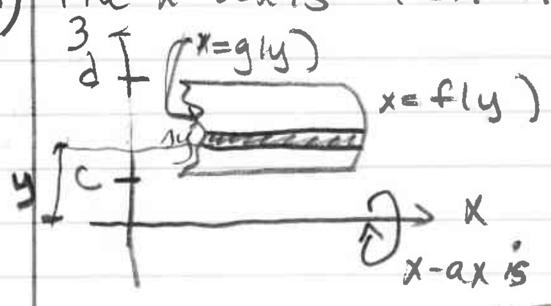
a) The y-axis (Washer method)



$$\Delta V = \pi\left([f(y)]^2 - [g(y)]^2\right)\Delta y$$

$$\boxed{V = \pi\int_c^d \left([f(y)]^2 - [g(y)]^2\right)dy}$$

b) The x-axis (Shell method)

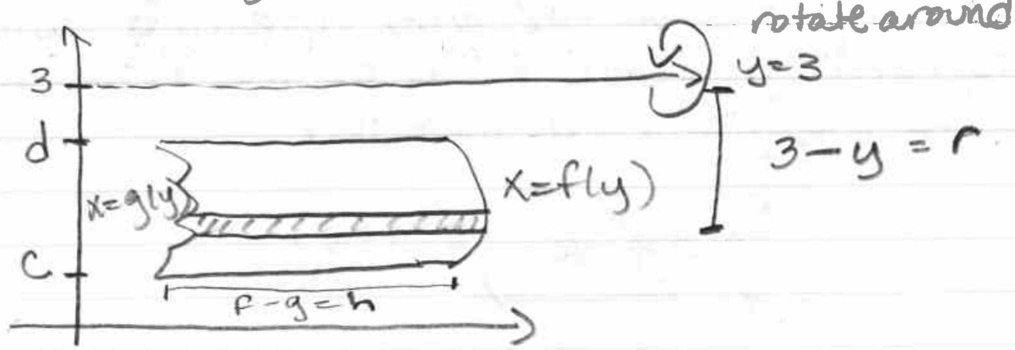


$$V = 2\pi r \cdot h \cdot \Delta r$$

$$\Delta V = 2\pi y\left(f(y) - g(y)\right)\Delta y$$

$$\boxed{V = 2\pi\int_c^d y[f(y) - g(y)]\,dy}$$

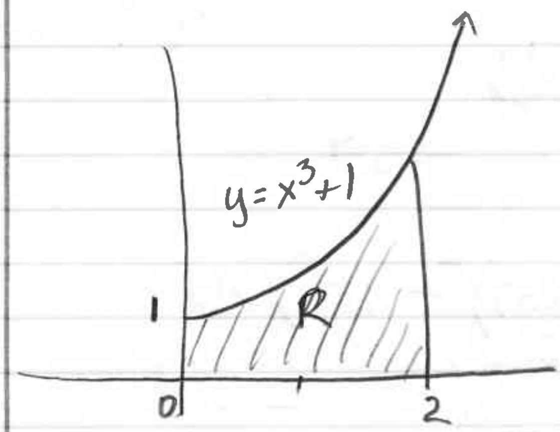
c) The line $y = 3$ (shell Method)



$$\Delta V = 2\pi r h \Delta r$$
$$= 2\pi(3-y)(f(y) - g(y))\Delta y$$

$$\boxed{V = 2\pi \int_c^d (3-y)(f(y)-g(y))\,dy}$$

6.3 (16) Sketch the region R bounded by $y=x^3+1$ and $y=0$ and between $x=0$ and $x=2$. Set up (but do not evaluate) integrals for each of the following.



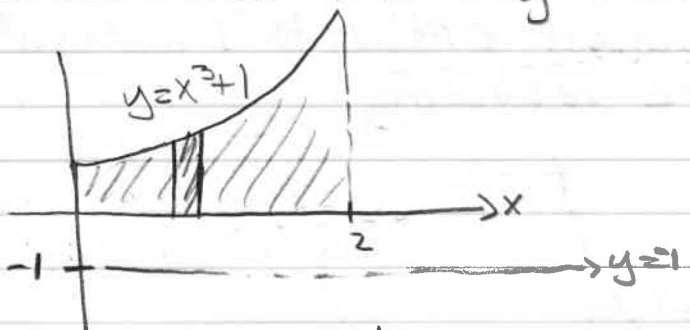
(a) $\boxed{\text{Area} = \int_0^2 (x^3+1)\,dx}$

(b) Volume of solid obtained when R is rotated about y-axis.

(shell method) $\Delta V = 2\pi r h \Delta r$
$= 2\pi x (x^3+1)\Delta x$

$$\boxed{V = 2\pi \int_0^2 x(x^3+1)\,dx = 2\pi \int_0^2 (x^4+x)\,dx}$$

(c) Volume of solid obtained when R is revolved about $y=-1$



Washer method.

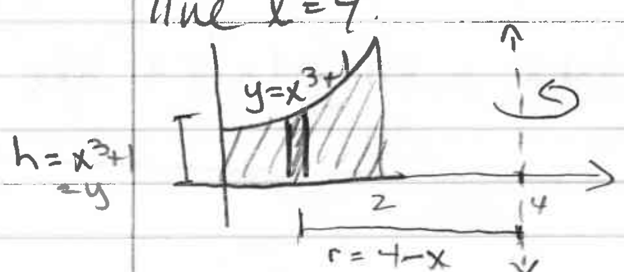
$$\Delta V = \pi \left(\left(x^3+1-(-1)\right)^2 - 1^2 \right) \Delta x$$

$$V = \pi \int_0^2 \left((x^3+2)^2 - 1^2 \right) dx$$

$$= \pi \int_0^2 \left(x^6 + 4x^3 + 4 - 1 \right) dx$$

$$\boxed{V = \pi \int_0^2 \left(x^6 + 4x^3 + 3 \right) dx}$$

d) Volume of solid obtained when R is revolved about line $x=4$.



Shell method.

$$\Delta V = 2\pi r \, h \, \Delta r$$

$$= 2\pi (4-x)(x^3+1)\Delta x$$

$$\boxed{V = 2\pi \int_0^2 (4-x)(x^3+1)\,dx = 2\pi \int_0^2 \left(-x^4 + 4x^3 - x + 4\right) dx}$$