5.6/4 Approximate the definite integral using $n=8$ and the following methods. Then calculate the definite integral exactly

$$\int_1^3 x\sqrt{x^2+1}\,dx \qquad \Delta x = h = \frac{b-a}{n} = \frac{3-1}{8}$$

$$\boxed{h = .25}$$

$x_0 = a = 1 \qquad f(x_0) = f(1) = 1\sqrt{1^2+1} = \sqrt{2} \approx 1.4142$

$x_1 = 1 + \Delta x = 1.25 \qquad f(1.25) = 1.25\sqrt{(1.25)^2+1} = 2.0010$

$x_2 = 1.50 \qquad f(1.5) = 1.5\sqrt{(1.5)^2+1} \approx 2.7042$

$x_3 = 1.75 \qquad f(1.75) = 1.75\sqrt{(1.75)^2+1} \approx 3.5272$

$x_4 = 2.0 \qquad f(2) = 2\sqrt{2^2+1} \approx 4.4721$

$x_5 = 2.25 \qquad f(2.25) = 2.25\sqrt{(2.25)^2+1} \approx 5.5400$

$x_6 = 2.50 \qquad f(2.5) = 2.5\sqrt{(2.5)^2+1} \approx 6.7315$

$x_7 = 2.75 \qquad f(2.75) = 2.75\sqrt{(2.75)^2+1} \approx 8.0470$

$x_8 = b = 3 \qquad f(3) = 3\sqrt{3^2+1} = 9.4868$

a) Left Riemann sum

$$\int_1^3 x\sqrt{x^2+1}\,dx \approx \Delta x\left[f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)+f(x_6)+f(x_7)\right]$$

$$\approx 0.25(1.4142+2.0010+2.7042+3.5272+4.4721+5.5400+6.7315+8.0470)$$

$$\approx \boxed{8.6093}$$

b) Right Riemann Sum

$$\int_1^3 x\sqrt{x^2+1}\,dx \approx \Delta x\left[f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)+f(x_6)+f(x_7)+f(x_8)\right]$$

$$\approx 0.25(2.0010+2.7042+3.5272+4.4721+5.5400+6.7315+8.0470+9.4868)$$

$$\approx \boxed{10.6274}$$

5.6/4 Continued..

c) Trapezoidal Rule $h = \Delta x$

$$\int_1^3 x\sqrt{x^2+1}\,dx \approx \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{7} f(x_i) + f(x_8)\right]$$

$$\approx \frac{.25}{2}\left[1.4142 + 2(2.0010 + 2.7042 + 3.5272 + 4.4721 + 5.54 + 6.7315 + 8.0470) + 9.4868\right]$$

$$\approx 9.6184$$

d) Parabolic Rule:

$$\int_1^3 x\sqrt{x^2+1}\,dx \approx \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_7) + f(x_8)\right]$$

$$\approx \frac{0.25}{3}\left[1.4142 + 4(2.0010) + 2(2.7042) + 4(3.5272) + 2(4.4721) + 4(5.5400) + 2(6.7315) + 4(8.0470) + 9.4868\right]$$

$$\approx \boxed{9.5981}$$

Exact Value (Fundamental Thm of Calculus)

$$\int_1^3 x\sqrt{x^2+1}\,dx$$

let $u = x^2+1$

$du = 2x\,dx$

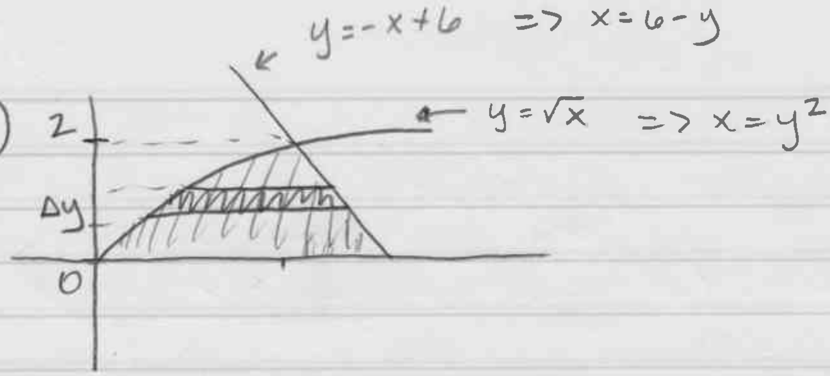
$x=1 \quad u = 1^2+1 = 2$

$x=3 \quad u = 3^2+1 = 10$

$$= \frac{1}{2}\int_2^{10} u^{1/2}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_2^{10}$$

$$= \frac{1}{3}\left[\sqrt{1000} - \sqrt{8}\right] \approx \boxed{9.5981}$$

6.1 10



functions

$y = -x+6 \longrightarrow x = 6-y$ set equal to find

$y = \sqrt{x} \Longrightarrow x = y^2$ points of intersection

$$y^2 = 6-y$$
$$y^2+y-6=0$$
$$(y+3)(y-2)=0$$

$y = -3$ not in range exclude

$y = 2$

$$\int_0^2 \left[(6-y) - y^2\right] dy = 6y - \frac{y^2}{2} - \frac{y^3}{3} \Big|_0^2$$

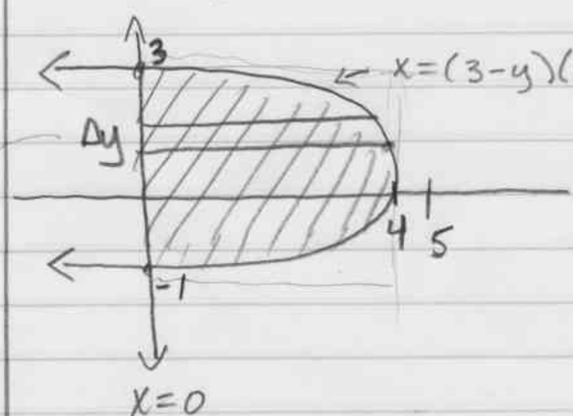
$$= 12 - 2 - \frac{8}{3} - 0$$

$$= \boxed{\frac{22}{3}}$$

6.1 24 Sketch the region bounded by the graphs, show a typical slice, approximate its area, set up an integral, and calculate the area of the region. Make an estimate to confirm your answer.

$x=(3-y)(y+1) \qquad x=0$

← equations in terms of y ∴ horizontal slices easier.



$x=0$ when
$(3-y)(y+1)=0$
$y=3, \ y=-1$
limits of integration
pts of intersection of $x=(3-y)(y+1)$ and $x=0$.

$$\Delta \text{Area} = \Delta A = (3-y)(y+1)\Delta y = (-y^2+2y+3)\Delta y$$

$$\text{Area} = \int_{-1}^{3}(-y^2+2y+3)\,dy = \left.\frac{-y^3}{3}+y^2+3y\right|_{-1}^{3}$$

$$\overset{F(3)}{=\left(\frac{-27}{3}+9+9\right)} - \overset{F(-1)}{\left(\frac{1}{3}+1-3\right)}$$

$$= 9 - \frac{1}{3} + 2$$

$$\boxed{\text{Area} = 10\tfrac{2}{3} \quad \text{OR} \quad \frac{32}{3}}$$

reasonable Estimate $\approx L\cdot W \approx 4\cdot 3 = 12$.
(draw box around graph and provide estimated dimensions.)