6.1 (34) An object moves along a line so that its velocity at time t is $v(t) = \frac{1}{2} + \sin(2t)$ ft/sec. Find the displacement and total distance traveled by the object for $0 \le t \le \frac{3\pi}{2}$

$$\text{Displacement} = \int_a^b v(t)\,dt = \int_0^{3\pi/2} \left(\tfrac{1}{2} + \sin(2t)\right) dt$$

$$= \int_0^{3\pi/2} \tfrac{1}{2}\,dt + \tfrac{1}{2}\int_0^{3\pi/2} 2\sin(2t)\,dt$$

$u = 2t$ substitution
$du = 2\,du$

$$= \int_0^{3\pi/2} \tfrac{1}{2}\,dt + \tfrac{1}{2}\int_0^{3\pi} \sin(u)\,du$$

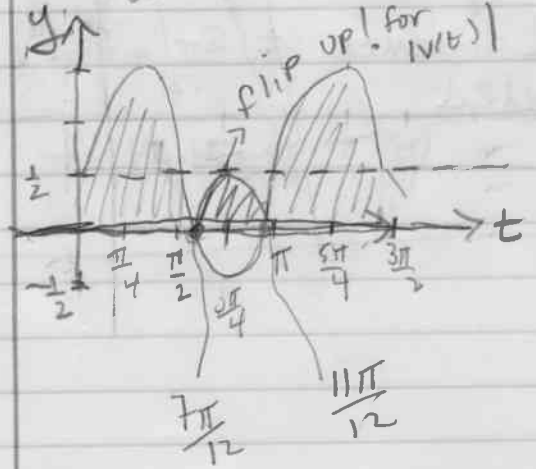
$$= \tfrac{1}{2}t\Big|_0^{3\pi/2} - \tfrac{1}{2}\cos u\Big|_0^{3\pi}$$

$$= \frac{3\pi}{4} - 0 - \left[\frac{-1}{2} - \frac{1}{2}\right] = \frac{3\pi}{4} + 1 \approx 3.36 \text{ ft}$$

= displacement where object is after $3\pi/2$ seconds.

TOTAL DISTANCE TRAVELED

$$= \int_a^b |v(t)|\,dt = \int_0^{3\pi/2}$$



$$\frac{1}{2} + \sin(2t) = 0$$

$$\sin(2t) = -\frac{1}{2}$$

$$2t = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$t = \frac{7\pi}{12}, \frac{11\pi}{12}$$

(over).

OR $\left|\frac{1}{2}+\sin 2t\right| = \begin{cases} \left(\frac{1}{2}+\sin 2t\right) & 0 \le t \le \frac{7\pi}{12} \text{ and } \frac{11\pi}{12} \le t \le \frac{3\pi}{2} \\ -\left(\frac{1}{2}+\sin 2t\right) & \frac{7\pi}{12} < t < \frac{11\pi}{12} \end{cases}$

↑ negate part of graph below x-axis to make it positive.

$$\therefore \int_0^{3\pi/2} |v(t)|\,dt = \int_0^{7\pi/12} \tfrac{1}{2}+\sin(2t)\,dt + \int_{7\pi/12}^{11\pi/12} -\tfrac{1}{2}-\sin(2t)\,dt + \int_{11\pi/12}^{3\pi/2} \tfrac{1}{2}+\sin(2t)\,dt$$

$$= \left(\tfrac{1}{2}t - \frac{\cos 2t}{2}\right)\Big|_0^{7\pi/12} + \left(-\tfrac{1}{2}t + \frac{\cos 2t}{2}\right)\Big|_{7\pi/12}^{11\pi/12} + \left(\tfrac{1}{2}t - \frac{\cos 2t}{2}\right)\Big|_{11\pi/12}^{3\pi/2}$$

$F(b)-F(a)$ $\quad$ $F(b)-F(a)$ $\quad$ $F(b)-F(a)$

$$\left(\frac{7\pi}{24}+\frac{\sqrt{3}}{4}+\frac{1}{2}\right) + \left(\frac{-11\pi}{24}+\frac{\sqrt{3}}{4}+\frac{7\pi}{24}+\frac{\sqrt{3}}{4}\right) + \left(\frac{3\pi}{4}+\frac{1}{2}-\frac{11\pi}{24}+\frac{\sqrt{3}}{4}\right)$$

$+\sqrt{3}$ $\qquad$ $+1$

$$= \frac{5\pi}{12}+\sqrt{3}+1 \approx 4.04 \text{ feet}$$

$\therefore \int_a^b |v(t)|\,dt$ = total distance traveled $= \frac{5\pi}{12}+\sqrt{3}+1 \approx 4.04$ ft