9.6 | Power series

(2) Find the convergence set for the given power series. $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$

By Absolute ratio test, if $\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$

the series converges.

$$\rho = \lim_{n\to\infty} \left| \frac{x^{n+1}}{3^{n+1}} \div \frac{x^n}{3^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x^{n+1}}{3^{n+1}} \cdot \frac{3^n}{x^n} \right| = \lim_{n\to\infty} \frac{|x|}{3} < 1$$

$$\Rightarrow \quad |x| < 3$$

$$\Rightarrow \quad -3 < x < 3$$

test convergence of series at endpoints.

at $x = -3$ $\quad \sum \frac{x^n}{3^n} = \sum \frac{(-3)^n}{3^n} = \sum (-1)^n$

alternating but $\lim a_n \neq 0$.

$\therefore$ series diverges

at $x = 3$ $\quad \sum \frac{x^n}{3^n} = \sum \frac{3^n}{3^n} = \sum (1)^n = 1+1+1+\cdots$

clearly divergent.

So convergence set of the series is

$$\boxed{\begin{array}{c} -3 < x < 3 \\ \text{or} \quad x \in (-3, 3) \end{array}}$$

9.6/power series

(10) Find the convergence set for the given power series.

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$= \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \qquad (0! = 1,\ 1! = 1)$$

$$a_n = \frac{x^n}{n!}$$ Find convergence set using Absolute RATIO TEST

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)!} \div \frac{x^n}{n!}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x^n \cdot x^1}{(n+1)\cdot n!} \cdot \frac{n!}{x^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x}{n+1}\right|$$

$$= \lim_{n\to\infty} |x| \cdot \left|\frac{1}{n+1}\right| = |x| \cdot 0 = 0 < 1$$

$\boxed{\therefore \text{ series converges for all } x.}$

$\mathbb{R}$ = convergence set

$-\infty < x < \infty$

$x \in (-\infty, \infty)$

9.6/ Power series.

(26) Find the convergence set for the power series.

$$\frac{(x-2)}{1^2} + \frac{(x-2)^2}{2^2} + \frac{(x-2)^3}{3^2} + \frac{(x-2)^4}{4^2} + \cdots \frac{(x-2)^n}{n^2}$$

$$a_1 + a_2 + a_3 + a_4 + \cdots a_n$$

$a_n = \dfrac{(x-2)^n}{n^2}$ Find convergence set for $\sum_{n=1}^{\infty} a_n$

using the Absolute RATIO Test and then testing endpoints for convergence.

$$\rho = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{(n+1)^2} \div \frac{(x-2)^n}{n^2}\right|$$

$$= \lim_{n\to\infty}\left|\frac{\cancel{(x-2)^n}\cdot(x-2)^1}{(n+1)^2}\cdot\frac{n^2}{\cancel{(x-2)^n}}\right|$$

$$= \lim_{n\to\infty}|x-2|\left(\frac{n}{n+1}\right)^2 = |x-2|$$

for convergence $\rho = |x-2| < 1$

$$|x-2| < 1$$

$$\Rightarrow \underset{+2}{-1} < \underset{+2}{x-2} < \underset{+2}{1}$$

$$1 < x < 3$$

when $x=1$ $a_n = \dfrac{(1-2)^n}{n^2} = \dfrac{(-1)^n}{n^2}$ alternating series

and $\lim a_n = 0$

series converges at $x=1$

when $x=3$ $a_n = \dfrac{(3-2)^n}{n^2} = \dfrac{1}{n^2}$ $\sum \frac{1}{n^2}$ converges by p-series test

$\therefore$ convergence set includes both endpts. $1 \le x \le 3$

$x \in [1,3]$