9.7/ Operations on Power series

Find the power series representation for $f(x)$ and specify the radius of convergence. It is somehow related to a geometric series

(6) $f(x) = \frac{1}{3+2x} = \frac{1}{3(1+\frac{2x}{3})} = \frac{\frac{1}{3}}{1-(-\frac{2}{3}x)} = \frac{a}{1-r}$

$\frac{a}{1-r} = a + ar + ar^2 + ar^3 + \cdots$

for $a = \frac{1}{3}$, $r = -\frac{2}{3}x$

$= \frac{1}{3} + \frac{1}{3}\left(-\frac{2}{3}x\right) + \frac{1}{3}\left(-\frac{2}{3}x\right)^2 + \frac{1}{3}\left(-\frac{2}{3}x\right)^3 + \cdots$

$= \frac{1}{3} - \frac{1}{3}\left(\frac{2}{3}x\right) + \frac{1}{3}\left(\frac{2}{3}x\right)^2 - \frac{1}{3}\left(\frac{2}{3}x\right)^3 + \cdots$

power series $\longrightarrow$ $= \frac{1}{3} - \frac{2}{9}x + \frac{4}{27}x^2 - \frac{8}{81}x^3 + \cdots$

radius of convergence when $|r| < 1$

$\left|-\frac{2}{3}x\right| < 1$

$\frac{2}{3}|x| < 1$

$|x| < \frac{3}{2}$

radius of conv $= \frac{3}{2}$

(14) Find the power series in x for the given function

$$f(x) = xe^{x^2}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\therefore e^{x^2} = 1 + (x^2) + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \cdots$$

$$= 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \cdots$$

$$\therefore xe^{x^2} = x\left(1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \cdots\right)$$

$$\boxed{xe^{x^2} = x + x^3 + \frac{x^5}{2!} + \frac{x^6}{3!} + \cdots}$$