9.2 Infinite Series

Indicate whether the series converges or diverges

If it converges, find its sum.

① $\sum_{k=1}^{\infty} \left(\frac{1}{7}\right)^k = \sum_{k=1}^{\infty} \frac{1}{7} \cdot \left(\frac{1}{7}\right)^{k-1} = \sum_{k=1}^{\infty} ar^{k-1}$

where $a = \frac{1}{7}$

$r = \frac{1}{7}$

since $|r| = \left|\frac{1}{7}\right| < 1$

the series converges to $\frac{a}{1-r}$

$\therefore \sum_{k=1}^{\infty} \left(\frac{1}{7}\right)^k = \frac{\frac{1}{7}}{1-\frac{1}{7}} = \frac{1}{7} \div \frac{6}{7} = \frac{1}{7} \cdot \frac{7}{6} = \boxed{\frac{1}{6}}$

② $\sum_{k=1}^{\infty} \left(-\frac{1}{4}\right)^{-k-2} = \left(-\frac{1}{4}\right)^{-3} + \left(-\frac{1}{4}\right)^{-4} + \left(-\frac{1}{4}\right)^{-5} + \cdots$

$= \left[\left(-\frac{1}{4}\right)^{-1}\right]^3 + \left[\left(-\frac{1}{4}\right)^{-1}\right]^4 + \left[\left(-\frac{1}{4}\right)^{-1}\right]^5 + \cdots$

$= (-4)^3 + (-4)^4 + (-4)^5 + \cdots$

$= (-4)^3 \cdot 1 + (-4)^3 \cdot (-4)^1 + (-4)^3 (-4)^2 + \cdots$

$= a \cdot r^{k-1} + a \cdot (-4)^1$

$a = (-4)^3 \quad r = -4 \quad |r| > 1$

$\therefore \sum_{k=1}^{\infty} \left(-\frac{1}{4}\right)^{-k-2}$

diverges.

9.2 Infinite Series.

Determine if series converges or diverges.

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$$\sum_{k=1}^{\infty}\left[5\left(\tfrac{1}{2}\right)^{k} - 3\left(\tfrac{1}{7}\right)^{k+1}\right]$$

$$= \sum_{k=1}^{\infty} 5\left(\tfrac{1}{2}\right)^{k} - \sum_{k=1}^{\infty} 3\left(\tfrac{1}{7}\right)^{k+1}$$

$$= \sum_{k=1}^{\infty} 5\cdot\tfrac{1}{2}\cdot\left(\tfrac{1}{2}\right)^{k-1} - \sum_{k=1}^{\infty} 3\cdot\left(\tfrac{1}{7}\right)^{2}\left(\tfrac{1}{7}\right)^{k-1}$$

$$= \sum_{k=1}^{\infty} \tfrac{5}{2}\left(\tfrac{1}{2}\right)^{k-1} - \sum_{k=1}^{\infty} \tfrac{3}{49}\left(\tfrac{1}{7}\right)^{k-1}$$

$a=\frac{5}{2}$ $r=\frac{1}{2}$

converges to

$$= \frac{a}{1-r} = \frac{\frac{5}{2}}{1-\frac{1}{2}} = \frac{\frac{5}{2}}{\frac{1}{2}} = \frac{5}{2}\cdot\frac{2}{1} = 5$$

$a=\frac{3}{49}$ $r=\frac{1}{7}$

converges to

$$\frac{a}{1-r} = \frac{\frac{3}{49}}{1-\frac{1}{7}} = \frac{\frac{3}{49}}{\frac{6}{7}} = \frac{\cancel{3}}{\cancel{49}_{7}}\cdot\frac{\cancel{7}}{\cancel{6}_{2}} = \frac{1}{14}$$

SUBTRACT

$$\therefore \sum_{k=1}^{\infty}\left[5\left(\tfrac{1}{2}\right)^{k} - 3\left(\tfrac{1}{7}\right)^{k+1}\right] = 5 - \frac{1}{14}$$

$$= \frac{70}{14} - \frac{1}{14} = \boxed{\frac{69}{14}}$$

9.2 / Infinite Series.

(7) $\sum_{k=2}^{\infty}\left(\frac{1}{k}-\frac{1}{k-1}\right) =$

CANCELS OUT

$\left(\frac{1}{2}-\frac{1}{1}\right)+\left(\frac{1}{3}-\frac{1}{2}\right)+\left(\frac{1}{4}-\frac{1}{3}\right)+\cdots$

$S_n = \left(\frac{1}{2}-1\right)+\left(\frac{1}{3}-\frac{1}{2}\right)\cdots+\left(\frac{1}{n-1}-\frac{1}{n-2}\right)+\left(\frac{1}{n}-\frac{1}{n-1}\right)$

CANCELS OUT　　　CANCELS OUT

$S_n = -1+\frac{1}{n}$ (only terms that don't "cancel")

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} -1+\frac{1}{n} = -1+0 = -1$

$\therefore \sum_{k=2}^{\infty}\left(\frac{1}{k}-\frac{1}{k-1}\right) = \boxed{-1}$

9.2

(16) Write the given decimal as an infinite series, then find the sum of the series, and finally, use the result to write the decimal as a ratio of 2 integers

$$0.\underline{\underline{21}}212121\ldots = .21 + .0021 + .000021 + .00000021 + \ldots$$

$$= \frac{21}{100} + \frac{21}{100}\cdot\left(\frac{1}{100}\right)^1 + \frac{21}{100}\left(\frac{1}{100}\right)^2 + \frac{21}{100}\left(\frac{1}{100}\right)^3 + \ldots$$

$$= \sum_{k=1}^{\infty} \frac{21}{100}\left(\frac{1}{100}\right)^{k-1}$$

$$a = \frac{21}{100} \quad r = \frac{1}{100} \quad |r| < 1$$

$\therefore$ $\sum ar^k$ converges

$$= \frac{a}{1-r}$$

$$= \frac{\frac{21}{100}}{1 - \frac{1}{100}}$$

$$= \frac{\frac{21}{100}}{\frac{99}{100}} = \frac{21}{1\cancel{00}} \cdot \frac{1\cancel{00}}{99}$$

$$= \frac{21}{99} = \frac{\cancel{3}\cdot 7}{\cancel{3}\cdot 33} = \boxed{\frac{7}{33}}$$