9.1 / Infinite sequences

(6) Write the first five terms of the sequence $\{a_n\}$. Determine if the sequence converges or diverges. If it converges, find $\lim_{n\to\infty} a_n$.

$$a_n = \frac{\sqrt{3n^2+2}}{2n+1}$$

$n=1 \quad a_1 = \frac{\sqrt{3+2}}{2+1} = \frac{\sqrt{5}}{3}$

$n=2 \quad a_2 = \frac{\sqrt{12+2}}{4+1} = \frac{\sqrt{14}}{5}$

$n=3 \quad a_3 = \frac{\sqrt{27+2}}{6+1} = \frac{\sqrt{29}}{7}$

$n=4 \quad a_4 = \frac{\sqrt{48+2}}{8+1} = \frac{\sqrt{50}}{9} = \frac{5\sqrt{2}}{9}$

$n=5 \quad a_5 = \frac{\sqrt{75+2}}{10+1} = \frac{\sqrt{77}}{11}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sqrt{3n^2+2}}{2n+1} = \lim_{n\to\infty} \frac{\sqrt{n^2}\sqrt{3+2/n^2}}{n(2+1/n)}$$

$$= \lim_{n\to\infty} \frac{n}{n} \cdot \frac{\sqrt{3+2/n^2}}{(2+1/n)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{3+2/n^2}}{(2+1/n)}$$

$$= \frac{\sqrt{3+0}}{2+0} = \boxed{\frac{\sqrt{3}}{2}}$$

converges

9.1 Infinite sequences

(8) Find the first five terms of the sequence $\{a_n\}$ and find $\lim\limits_{n\to\infty} a_n$ if it converges.
and state sequence diverges if $\lim\limits_{n\to\infty} a_n$ doesn't exist.

$$a_n = \frac{n\cos(n\pi)}{2n-1}$$

$$a_1 = \frac{1\cdot\cos(\pi)}{2\cdot1-1} = \frac{-1}{1} = -1$$

$$a_2 = \frac{2\cos(2\pi)}{2\cdot2-1} = \frac{2\cdot1}{3} = \frac{2}{3}$$

$$a_3 = \frac{3\cos(3\pi)}{2\cdot3-1} = \frac{3\cdot(-1)}{5} = \frac{-3}{5}$$

$$a_4 = \frac{4\cos(4\pi)}{2\cdot4-1} = \frac{4\cdot1}{7} = \frac{4}{7}$$

$$a_5 = \frac{5\cos(5\pi)}{2\cdot5-1} = \frac{5\cdot(-1)}{9} = -\frac{5}{9}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n\cdot\cos(n\pi)}{2n-1} = \lim_{n\to\infty}\frac{n}{2n-1}\cdot\lim_{n\to\infty}\cos(n\pi)$$

$$\lim_{n\to\infty}\cos(n\pi) = \begin{cases} 1 & n = \text{even} \\ -1 & n = \text{odd.} \end{cases}$$

$$\text{and } \lim_{n\to\infty}\frac{n}{2n-1} = \lim_{n\to\infty}\frac{n}{n}\cdot\frac{1}{2-\frac{1}{n}} = \lim_{n\to\infty}\frac{1}{2-\frac{1}{n}} = \frac{1}{2}$$

$$\therefore \lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n}{2n-1}\cdot\lim_{n\to\infty}\cos(n\pi) \quad \boxed{\text{diverges}}$$

it oscillates between 1/2 and -1/2 limit doesn't exist!

9.1/ Infinite Sequences

(18) Write the first five terms of the sequence $\{a_n\}$ and determine if it converges or diverges. If convergent find $\lim_{n\to\infty} a_n$

$$a_n = \frac{\ln(1/n)}{\sqrt{2n}}$$

$$\boxed{\begin{array}{l}\text{converges to } 0 \\ a_n \longrightarrow 0 \quad \text{as} \quad n \to \infty\end{array}}$$

$$a_1 = \frac{\ln(1/1)}{\sqrt{2\cdot 1}} = 0$$

$$a_2 = \frac{\ln(1/2)}{\sqrt{2\cdot 2}} = \frac{\ln(1/2)}{2} = -0.3466$$

$$a_3 = \frac{\ln(1/3)}{\sqrt{2\cdot 3}} = \frac{\ln(1/3)}{\sqrt{6}} = -0.4485$$

$$a_4 = \frac{\ln(1/4)}{\sqrt{2\cdot 4}} = \frac{\ln(1/4)}{\sqrt{8}} = -0.4901$$

$$a_5 = \frac{\ln(1/5)}{\sqrt{2\cdot 5}} = \frac{\ln(1/5)}{\sqrt{10}} = -0.5089$$

Consider $\lim_{x\to\infty} \frac{\ln(1/x)}{\sqrt{2x}} = \lim_{x\to\infty} \frac{\ln(x^{-1})}{\sqrt{2x}} = \lim_{x\to\infty} \frac{-\ln x}{\sqrt{2x}}$

L'Hopital's Rule applies $\left(\frac{\infty}{\infty}\right)$

$$\therefore \lim_{x\to\infty} \frac{-\ln x}{(2x)^{1/2}} = \lim_{x\to\infty} \frac{-\frac{1}{x}}{\frac{1}{2}\cdot(2x)^{-1/2}\cdot 2}$$

$$= \lim_{x\to\infty} \frac{\frac{-1}{x}}{\frac{1}{\sqrt{2x}}} = \lim_{x\to\infty} \frac{-\sqrt{2x}}{x}$$

$$= \lim_{x\to\infty} \frac{-\sqrt{2}\cdot\cancel{\sqrt{x}}}{\sqrt{x}\cdot\cancel{\sqrt{x}}}$$

$$= \lim_{x\to\infty} \frac{-\sqrt{2}}{\sqrt{x}} = 0$$

since $\lim_{x\to\infty} a(x) = 0$

$\lim_{n\to\infty} a_n = 0$

9.1 / Infinite sequences

(22) Find an explicit formula $a_n =$ ____ for the sequence. Determine whether the sequence converges or diverges. If it converges, find $\lim_{n\to\infty} a_n$.

$$\frac{1}{2^2}, \frac{2}{2^3}, \frac{3}{2^4}, \frac{4}{2^5}, \ldots, a_n, \ldots$$

$n=1$, $n=2$, $n=3$, $n=4$, ... $n=n$, ...

numerator obviously $= n$ for a_n.
denominator call D_n

n	D_n			
1	2^2	$\rightarrow$	2^{1+1}	$= 2^{n+1}$
2	2^3	$=$	2^{2+1}	$= 2^{n+1}$
3	2^4	$=$	2^{3+1}	$= 2^{n+1}$
4	2^5	$=$	2^{4+1}	$= 2^{n+1}$
n	2^{n+1}	$=$	2^{n+1}	$= 2^{n+1}$

$$\therefore a_n = \frac{\text{num}}{\text{den}} = \boxed{\frac{n}{2^{n+1}}}$$

Consider $\lim_{x\to\infty} \frac{x}{2^{x+1}} = \frac{1}{2}\lim_{x\to\infty}\frac{x}{2^x}$ L'Hopital's Rule applies $\left(\frac{\infty}{\infty}\right)$

$$= \frac{1}{2}\lim_{x\to\infty}\frac{x}{2^x} = \frac{1}{2}\lim_{x\to\infty}\frac{1}{\ln 2\cdot 2^x} = \frac{1}{2}\cdot\frac{1}{\ln 2}\cdot 0 = 0$$

$\therefore$ Since $\lim_{x\to\infty} a(x) = 0$, $\lim_{n\to\infty} a_n = 0$

$\boxed{\text{sequence } \{a_n\} = \left\{\frac{n}{2^{n+1}}\right\} \text{ converges to } 0.}$