9.5 / Alternating Series, Absolute Convergence, + Conditional Conv.

(2) Show the alternating series converges.
Estimate the error made by using the partial sum S_9 as an approximation to the sum S of the series.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} - \cdots$$

$$a_n = \frac{1}{\sqrt{n}} \quad \text{and} \quad \frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}} > 0$$

so $a_n > a_{n+1}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$$

$\therefore$ The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$ converges

by the Alternating Series Test

$$|S - S_9| \le \boxed{a_{10} = \frac{1}{\sqrt{10}} \approx .31623}$$

(8) Show the series converges absolutely.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n\sqrt{n}}$$

$$a_n = (-1)^n \frac{1}{n\sqrt{n}} \qquad |a_n| = \frac{1}{n\sqrt{n}}$$

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$ since $\frac{3}{2} > 1$

converges by the p-series test.

Since $\sum |a_n|$ converges $\sum (-1)^n \frac{1}{n\sqrt{n}}$ converges absolutely.

9.5 / Absolute convergence Conditional Convergence

(20) Classify the series as absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=2}^{\infty} (-1)^n \frac{1}{\sqrt{n^2-1}}$$

alternating series. $a_n = \frac{1}{\sqrt{n^2-1}}$ $\quad a_{n+1} = \frac{1}{\sqrt{n^2+2n}}$

because denominator is smaller $\frac{1}{\sqrt{n^2-1}} > \frac{1}{\sqrt{n^2+2n}} > 0$

so $\quad a_n > a_{n+1} > 0 \quad \forall n \geq 2$

$\therefore$ By the alternating series test the series converges.

Now consider $\sum |a_n| = \sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}}$

let $b_n = \frac{1}{n}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2-1}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2-1}} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2}\sqrt{1-\frac{1}{n^2}}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{1-\frac{1}{n^2}}} = 1$$

$0 < 1 < \infty$ $\quad \therefore$ By the limit comparison test since $\sum \frac{1}{n} = \sum b_n$ diverges

$$\sum |a_n| = \sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}} \text{ diverges}$$

Hence $\sum a_n$ converges but $\sum |a_n|$ diverges implies $\sum (-1)^n \frac{1}{\sqrt{n^2-1}}$ is conditionally convergent.