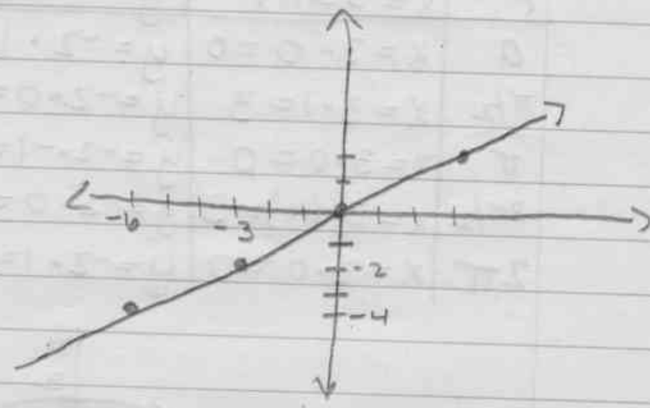
10.4/Parametric Representation of Curves in the Plane

(2) $x = 3t \quad y = 2t \quad -\infty < t < \infty$

(a) Graph the curve.

t	$x=3t$	$y=2t$	(x,y)
-2	$x=-6$	$y=-4$	$(-6,-4)$
-1	$x=-3$	$y=-2$	$(-3,-2)$
0	$x=0$	$y=0$	$(0,0)$
1	$x=3$	$y=2$	$(3,2)$



(b) Is the curve closed? Is it simple?
The curve is not closed since endpoints do not coincide.
The curve is simple because no two distinct values of t produce the same point in the plane (the curve does not cross, or contain "loops")

(c) Obtain the Cartesian equation of the curve by eliminating the parameter.
(easy to solve one parameter for t and substitute it into the other parameter.)

$\underline{x = 3t} \Rightarrow t = \frac{x}{3}$ $\qquad y = 2t$

$= 2\left(\frac{x}{3}\right)$

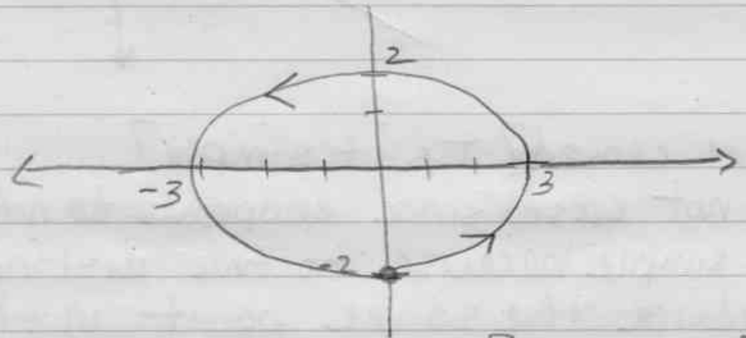
$\boxed{y = \frac{2}{3}x}$ line with slope $m = \frac{2}{3}$

10.4/ Parametric Representation of Curves in the Plane

(14) $x = 3\sin r \quad y = -2\cos r \quad ; \quad 0 \le r \le 2\pi$

(a) Graph the curve

r	$x = 3\sin r$	$y = -2\cos r$	(x, y)
0	$x = 3 \cdot 0 = 0$	$y = -2 \cdot 1 = -2$	$(0, -2)$
$\pi/2$	$x = 3 \cdot 1 = 3$	$y = -2 \cdot 0 = 0$	$(3, 0)$
π	$x = 3 \cdot 0 = 0$	$y = -2 \cdot -1 = 2$	$(0, 2)$
$3\pi/2$	$x = 3 \cdot (-1) = -3$	$y = -2 \cdot 0 = 0$	$(-3, 0)$
2π	$x = 3 \cdot 0 = 0$	$y = -2 \cdot 1 = -2$	$(0, -2)$



(b) Is the curve closed? Is it simple?

The curve is closed because initial and final end points are the same.

The curve is simple because no 2 distinct values of r in $0 < r < 2\pi$ (interior of interval only) produce the same output (no intersection pts except at the endpoints.)

(c) Find the rectangular/cartesian equation for the curve.

$x = 3\sin r \qquad y = -2\cos r$

$\frac{x}{3} = \sin r \qquad \frac{y}{-2} = \cos r$

$\frac{x^2}{9} = \sin^2 r \qquad \frac{y^2}{4} = \cos^2 r$

$\sin^2 r + \cos^2 r = 1$

$\boxed{\frac{x^2}{9} + \frac{y^2}{4} = 1}$

10.4/parametric Representation

(22) Find dy/dx and d^2y/dx^2 without eliminating the parameter.

$x = 6s^2 \qquad y = -2s^3 \; ; \; s \neq 0$

$\frac{dx}{ds} = 12s \qquad \frac{dy}{ds} = -6s^2$

$$\frac{dy}{dx} = \frac{dy/ds}{dx/ds} = \frac{-6s^2}{12s} = \boxed{-\frac{1}{2}s = \frac{dy}{dx}}$$

$$\frac{dy}{dx} = y' = -\frac{1}{2}s$$

$$\frac{dy'}{ds} = -\frac{1}{2}$$

$$\frac{d^2y}{dx^2} = \frac{dy'}{dx} = \frac{dy'/ds}{dx/ds} = \frac{-1/2}{12s} = \boxed{-\frac{1}{24s} = \frac{d^2y}{dx^2}}$$

10.4 / Parametric Representation of Curves in the Plane

(28) Find dy/dx and d^2y/dx^2 without eliminating the parameters.

$x = \cot t - 2 \qquad y = -2\csc t + 5 \qquad 0 < t < \pi$

$\frac{dx}{dt} = -\csc^2 t \qquad \frac{dy}{dt} = 2\csc t \cot t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\csc t \cot t}{-\csc^2 t} = -\frac{2\cot t}{\csc t}$$

$$= -\frac{2\cos t}{\sin t} \div \frac{1}{\sin t}$$

$$= -\frac{2\cos t}{\cancel{\sin t}} \cdot \frac{\cancel{\sin t}}{1}$$

$$\boxed{\frac{dy}{dx} = -2\cos t}$$

$\frac{dy}{dx} = y' = -2\cos t$

$\frac{dy'}{dt} = 2\sin t$

$$\frac{d^2y}{dx^2} = \frac{dy'}{dx} = \frac{dy'/dt}{dx/dt} = \frac{2\sin t}{-\csc^2 t}$$

$$= 2\sin t \div -\frac{1}{\sin^2 t}$$

$$= 2\sin t \cdot -\frac{\sin^2 t}{1}$$

$$\boxed{\frac{d^2y}{dx^2} = -2\sin^3 t}$$

10.4 / Parametric Representation of Curves in the Plane

(40) Find the length of the parametric curve defined over the given interval.

$x = t + \frac{1}{t}$ $\quad y = \ln t^2$ $\quad 1 \le t \le 4$

$$\frac{dx}{dt} = 1 - \frac{1}{t^2}$$

$$= \frac{t^2-1}{t^2}$$

$$\left(\frac{dx}{dt}\right)^2 = \left(\frac{t^2-1}{t^2}\right)^2 = \frac{(t^2-1)(t^2-1)}{t^4} = \frac{t^4-2t^2+1}{t^4}$$

$$\frac{dy}{dt} = \frac{1}{t^2} \cdot 2t = \frac{2}{t}$$

$$\left(\frac{dy}{dt}\right)^2 = \left(\frac{2}{t}\right)^2 = \frac{4}{t^2}$$

$$L = \int_1^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_1^4 \sqrt{\frac{t^4-2t^2+1}{t^4} + \frac{4}{t^2}}\,dt$$

$$= \int_1^4 \sqrt{\frac{t^4-2t^2+1+4t^2}{t^4}}\,dt$$

$$= \int_1^4 \sqrt{\frac{t^4+2t^2+1}{t^4}}\,dt$$

$$= \int_1^4 \sqrt{\frac{(t^2+1)^2}{t^4}}\,dt = \int_1^4 \frac{t^2+1}{t^2}\,dt$$

next line

$$= \int_1^4 1 + \frac{1}{t^2}\,dt = \int_1^4 (1+t^{-2})\,dt = t - \frac{1}{t}\Big|_1^4 = \left(4-\frac{1}{4}\right) - \left(1-\frac{1}{1}\right)$$

$$= \boxed{15/4}$$