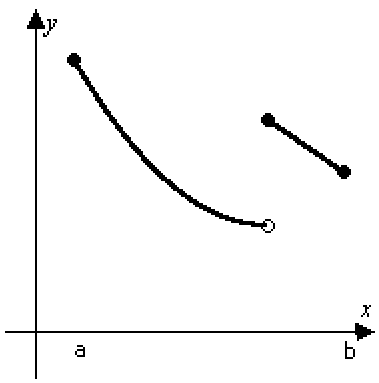
Name___________________________________

Determine from the graph whether the function has any absolute extreme values on the interval [a, b].

1) 1) ________



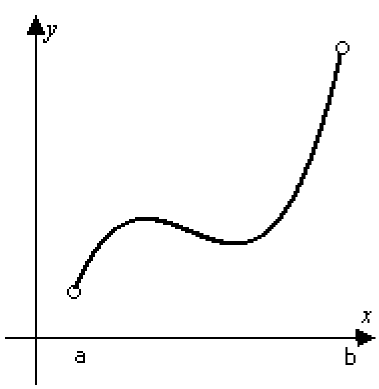
A) No absolute extrema.

B) Absolute maximum only.

C) Absolute minimum only.

D) Absolute minimum and absolute maximum.

2) 2) ________



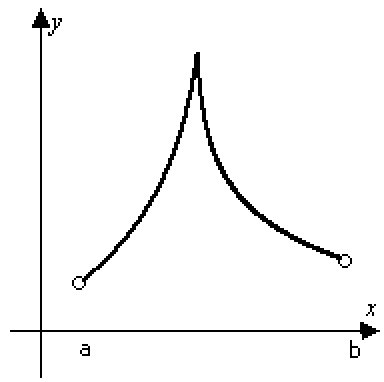
A) No absolute extrema.

B) Absolute maximum only.

C) Absolute minimum only.

D) Absolute minimum and absolute maximum.

3) 3) ________



A) No absolute extrema.

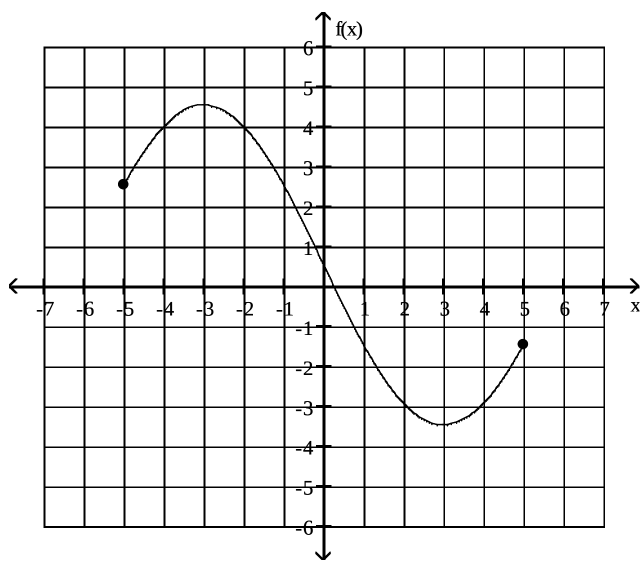
B) Absolute minimum only.

C) Absolute minimum and absolute maximum.

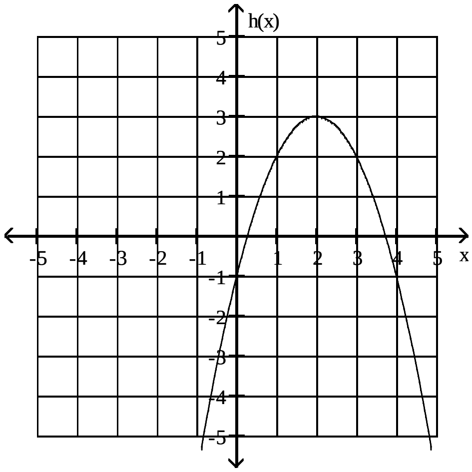
D) Absolute maximum only.

Find the location of the indicated absolute extremum for the function.

4) Minimum 4) ________________

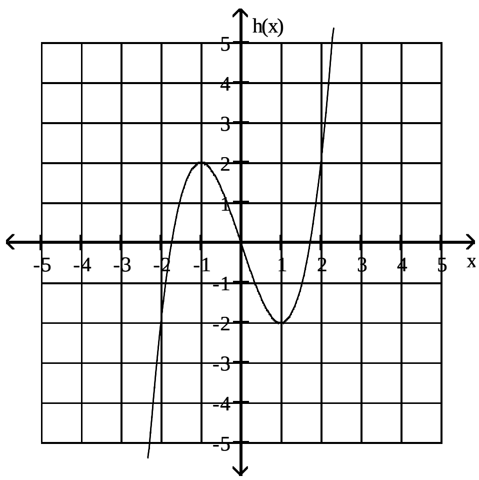


5) Maximum



5) ________________

6) Minimum



6) ________________

Identrify the critical points and find the maximum and minimum value on the given interval I.

7) $f(x) = x^2 + 18x + 81$; $I = [-18, 0]$

7) ________________

8) $f(x) = x^2 + 4x$; $I = \left[-\frac{5}{2}, -\frac{1}{2}\right]$

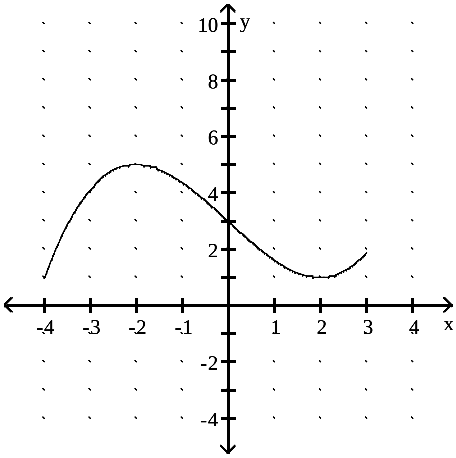
8) ________________

9) $f(r) = \frac{1}{r^2 + 2}$; $I = [-1, 5]$

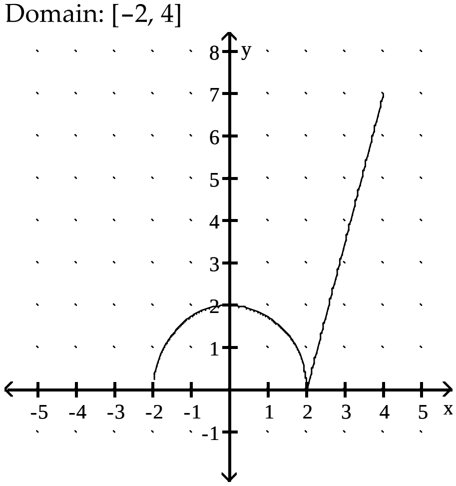
9) ________________

Find all critical points and find the minimum and maximum value of the function on the given domain.

10) Domain: [−4, 3] 10) ________________



11) Domain: [−2, 4] 11) ________________



Use the Concavity Theorem to determine where the given function is concave up and where it is concave down. Also find all inflection points.

12) $T(t) = 2t - t^3$ 12) ________________

13) $f(x) = x^3 + 3x^2 - x - 24$ 13) ________________

Use the Montonicity Theorem to find where the function is increasing and where it is decreasing.

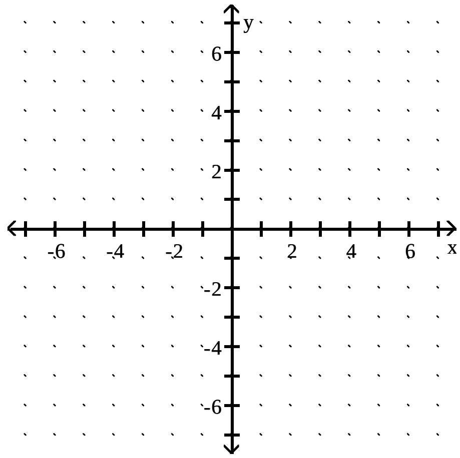
14) $g(x) = x^2 - 2x + 1$ 14) ________________

15) $h(t) = \dfrac{1}{t^2 + 1}$ 15) ________________

16) $h(t) = \cos t, 0 \leq t \leq 2\pi$ 16) _______________

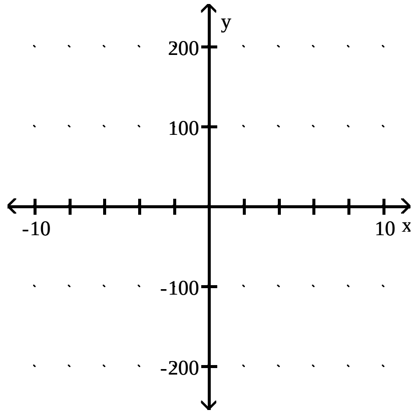
Sketch the graph of a continuous function f on the given domain that satisfies all conditions.

17) f has domain [0, 6]; $f(0) = 0$; $f(6) = 4$; increasing and concave down on (0, 6) 17) _______________

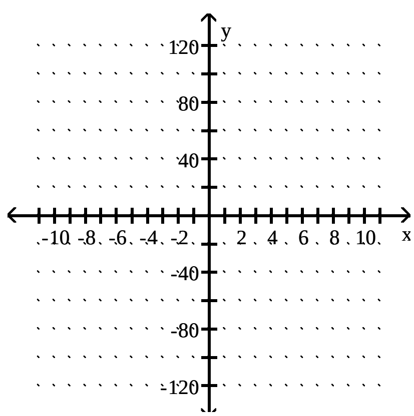


Determine where the graph of the function is increasing, decreasing, concave up, concave down. Then sketch the graph.

18) $f(x) = 9x^2 + 72x$ 18) _______________



19) $f(x) = 48x - x^3$ 19) _______________



The first derivative f ' is given. Find all values of x that make the function a local minimum and a local maximum.

20) $f'(x) = (x + 4)(x + 8)$ 20) ______________

Identify the critical points. Then use the test of your choice to decide which critical points give a local maximum value and which give a local minimum value. Give these values.

21) $f(x) = x^3 - 6x^2 + 4$ 21) ______________

22) $f(\theta) = \cos 2\theta,\ 0 < \theta < \frac{\pi}{4}$ 22) ______________

23) $g(x) = \frac{x}{x^2 + 1}$ 23) ______________

24) $f(x) = (x - 10)^3$ 24) ______________

Find, if possible, the (global) maximum and minimum values of the given function on the indicated interval.

25) $f(x) = x - 4$ on $[-2, 4]$ 25) ______________

Solve the problem.

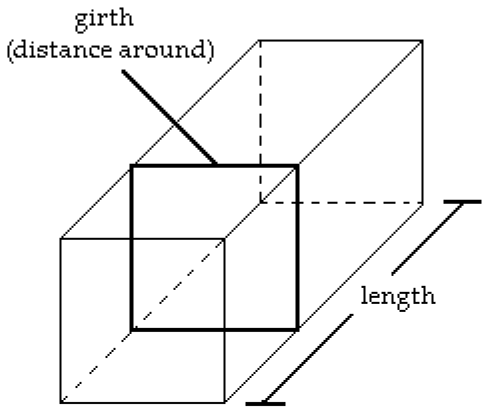
26) If the price charged for a candy bar is p(x) cents, then x thousand candy bars will be sold in a certain city, where $p(x) = 62 - \frac{x}{12}$. How many candy bars must be sold to maximize revenue? 26) ______________

27) Suppose a business can sell x gadgets for $p = 250 - 0.01x$ dollars apiece, and it costs the business $c(x) = 1000 + 25x$ dollars to produce the x gadgets. Determine the production level and the sale price per gadget required to maximize profit. 27) ______________

28) Find two numbers whose product is -81 and the sum of whose squares is a minimum. 28) ______________

29) A company wishes to manufacture a box with a volume of 20 cubic feet that is open on top and is twice as long as it is wide. Find the width of the box that can be produced using the minimum amount of material. Round to the nearest tenth, if necessary. 29) ______________

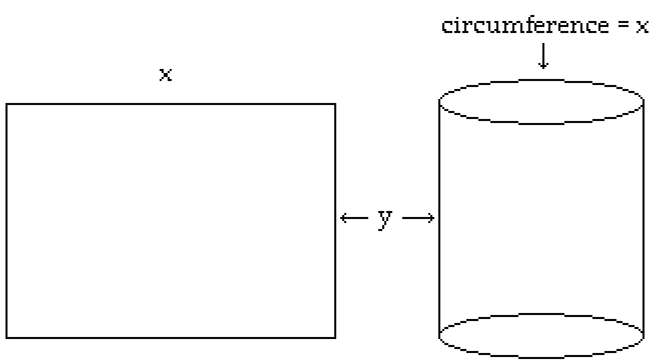
30) A private shipping company will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 120 inches. What dimensions will give a box with a square end the largest possible volume? 30) _______________



31) A rectangular field is to be enclosed on four sides with a fence. Fencing costs $5 per foot for two opposite sides, and $8 per foot for the other two sides. Find the dimensions of the field of area 750 square feet that would be the cheapest to enclose. 31) _______________

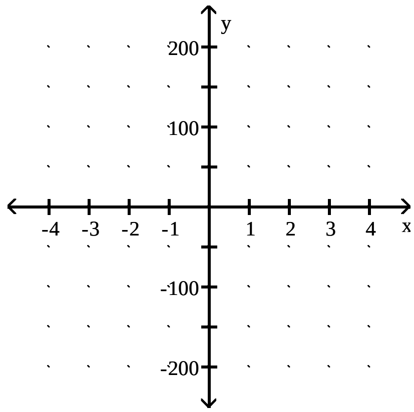
32) The velocity of a particle, in feet per second, is given by $v = t^2 - 8t + 2$, where t is the time (in seconds) for which it has traveled. Find the time at which the velocity is at a minimum. 32) _______________

33) A rectangular sheet of perimeter 39 centimeters and dimensions x centimeters by y centimeters is to be rolled into a cylinder as shown in the figure. What values of x and y give the largest volume? 33) _______________



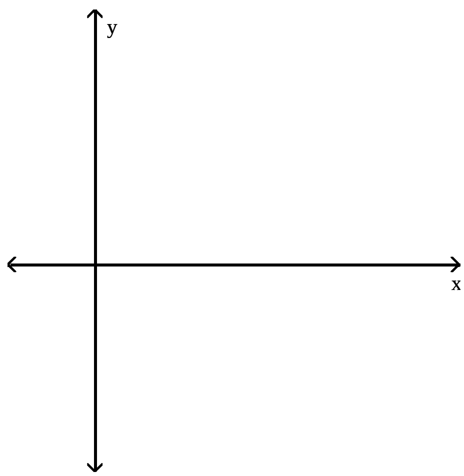
Make an analysis using calculus and sketch the graph.

34) $g(x) = 3x^4 - 12x^3$ 34) _______________



Sketch a possible graph of f(x) using f'(x).

35) $f'(x) = x^2(2 - x)$ and $f(0) = 0$ 35) _______________



Decide whether the Mean Value Theorem applies to the given function on the given interval.

36) $f(x) = x^{1/3}$; $[-3, 2]$ 36) _______________

Use the Mean Value Theorem and find all possible values of c on the given interval.

37) $f(x) = x^2 + 5x + 1$; $[-3, 2]$ 37) _______________

Solve the problem. Use a numerical method to approximate the solution.

38) The altitude h (in m) of a rocket is given by $h = -2t^3 + 90t^2 + 400t + 30$, where t is the time (in s) of the flight. When does the rocket hit the ground? 38) _______________

Find the indicated root of the given equation by using Newton's method.

39) $x^3 - x - 1 = 0$ (between 1 and 2) 39) _______________

Evaluate the indefinite integral.

40) $\int \left(2t^2 + \frac{t}{3}\right) dt$ 40) ______________

41) $\int \left(\frac{\sqrt{y}}{4} + \frac{3}{\sqrt{y}}\right) dy$ 41) ______________

42) $\int x^5(x^6 - 5)^4 \, dx$ 42) ______________

43) $\int \frac{\sin t}{(7 + \cos t)^5} \, dt$ 43) ______________

Find f(x) given f''(x). Your answer will involve two arbitrary constants.

44) $f''(x) = 7x + 6$ 44) ______________

Find the general antiderivative F(x) + C for the function.

45) $f(x) = 5x - 5$ 45) ______________

46) $f(x) = 3\sqrt{x} + 9$ 46) ______________

47) $f(x) = -\frac{32}{x^5}$ 47) ______________

Find the particular solution that satisfies the given condition.

48) $\frac{dy}{dx} = x - 6$; curve passes through (2, 5) 48) ______________

49) $\frac{dy}{dx} = 2x^{-3/4}$; curve passes through (1, 3) 49) ______________

Determine if the function is a solution of the differential equation.

50) $x\frac{dy}{dx} - y = 0$; $y = Cx$ 50) ______________

Use L'Hôpital's rule to find the limit.

51) $\lim_{x \to 9} \frac{x^2 - 81}{x - 9}$ 51) ______________

52) $\lim_{x \to 0} \frac{\cos 3x - 1}{x^2}$ 52) ______________

53) $\lim_{x \to \frac{\pi}{3}} \frac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}}$ 53) ______________

54) $\lim_{x \to 0} \frac{\sin (5x)}{\sin x}$ 54) ______________

55) $\lim_{x \to 0} \frac{\sin 2x}{\tan 10x}$ 55) ______________

56) $\lim_{x \to 0} \frac{e^x - 1}{\sin 7x}$ 56) ______________

57) $\lim_{x \to 0} \frac{\sin x}{7x + x^4}$ 57) ______________

58) $\lim_{x \to 0} \frac{5^x - 1}{7^x - 1}$ 58) ______________

59) $\lim_{x \to 16} \frac{e^x - e^{16}}{x - 16}$ 59) ______________

60) $\lim_{x \to \infty} \frac{15x^2 - 8x - 7}{10x^2 - 7x + 8}$ 60) ______________

61) $\lim_{x \to \infty} (\sqrt{x^2 + 7x} - x)$ 61) ______________

62) $\lim_{x \to 0^+} x^{\sin x}$ 62) ______________

63) $\lim_{x \to \infty} \frac{e^x}{4x^2 + 9}$ 63) ______________

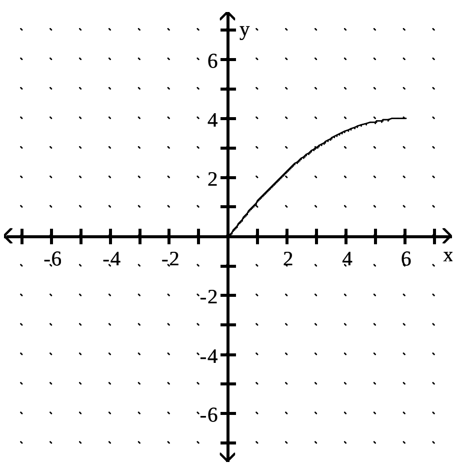
64) $\lim_{x \to \infty} \frac{3x^{576}}{e^x}$ 64) ______________

65) $\lim_{x \to 0^+} (x^2 \ln x)$ 65) ______________

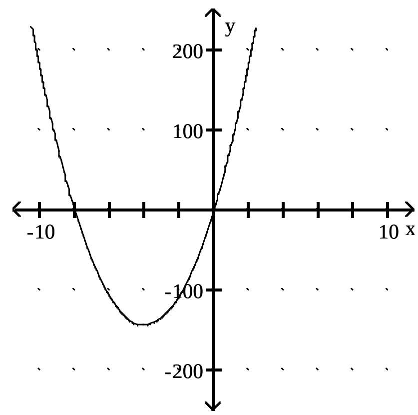
66) $\lim_{x \to 1} x^{1/(15x - 15)}$ 66) ______________

67) $\lim_{x \to \pi/2} \frac{\csc 10x}{\csc 8x}$ 67) ______________

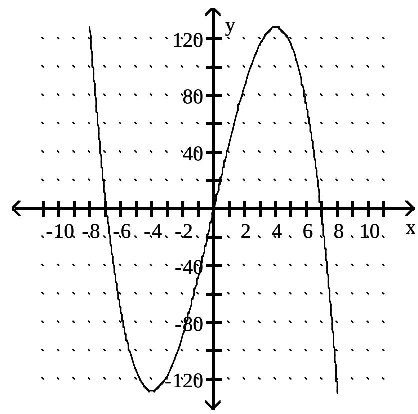
1) B
2) A
3) D
4) x = 3
5) x = 2
6) No minimum
7) Critical points: –18, –9, 0; maximum value 81; minimum value 0
8) Critical points: $-\frac{5}{2}$, – 2, $-\frac{1}{2}$; maximum value $-\frac{7}{4}$; minimum value – 4
9) Critical points: –1, 0, 5; maximum value $\frac{1}{2}$; minimum value $\frac{1}{27}$
10) Critical points: –4, –2, 2, 3; maximum value: 5; minimum value: 1
11) Critical points: –2, 0, 2, 4; maximum value: 7; minimum value: 0
12) Concave up on (–∞, 0), concave down on (0, ∞); inflection point (0, 0)
13) Concave up on (–1, ∞), concave down on (–∞, –1); inflection point (–1, –21)
14) Increasing on [1, ∞), decreasing on (–∞, 1]
15) Increasing on (–∞, 0], decreasing on [0, ∞)
16) Increasing on [π, 2π], decreasing on [0, π]
17)



18)

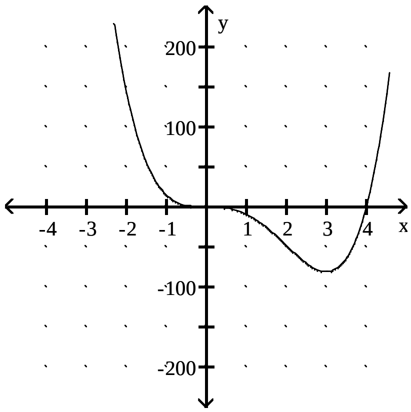


19)

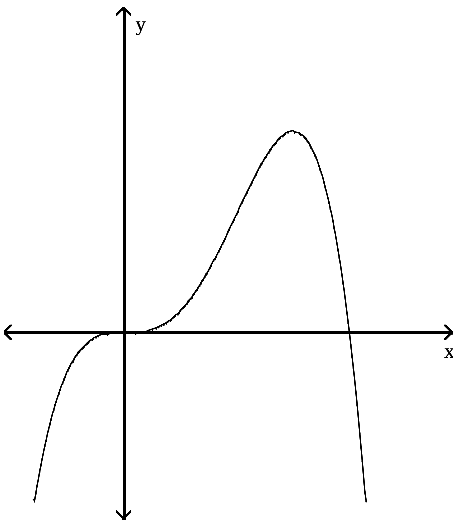


20) Local minimum at x = –4; local maximum at x = –8
21) Critical points: 0, 4; local maximum f(0) = 4; local minimum f(4) = –28
22) No critical points; no local maxima or minima on the interval $\left(0, \frac{\pi}{4}\right)$
23) Critical points: –1, 1; local maximum f(1) = $\frac{1}{2}$; local minimum f(–1) = $-\frac{1}{2}$
24) Critical point: 10; no local minima or maxima
25) Maximum value f(4) = 0; minimum value f(–2) = – 6
26) 372 thousand candy bars
27) 11,250 gadgets at $137.50 each
28) 9 and –9
29) 2.4 ft
30) 20 in. by 20 in. by 40 in.
31) 34.6 ft at $5 by 21.7 ft at $8
32) 4 sec
33) x = 13 cm; y = $\frac{13}{2}$ cm

34)



35)



36) No

37) $c = -\frac{1}{2} = -0.5$

38) 49.081 s

39) 1.3247180

40) $\frac{2}{3}t^3 + \frac{t^2}{6} + C$

41) $\frac{1}{6}y^{3/2} + 6\sqrt{y} + C$

42) $\frac{(x^6 - 5)^5}{30} + C$

43) $\frac{1}{4(7 + \cos t)^4} + C$

44) $f(x) = \frac{7}{6}x^3 + 3x^2 + C_1x + C_2$

45) $\frac{5}{2}x^2 - 5x + C$

46) $2x^{3/2} + 9x + C$

47) $\frac{8}{x^4} + C$

48) $y = \frac{x^2}{2} - 6x + 15$

49) $y = 8x^{1/4} - 5$

50) Yes

51) 18

52) $-\frac{9}{2}$

53) $-\frac{\sqrt{3}}{2}$

54) 5

55) $\frac{1}{5}$

56) $\frac{1}{7}$

57) $\frac{1}{7}$

58) $\frac{\ln 5}{\ln 7}$

59) e^{16}

60) $\frac{3}{2}$

61) $\frac{7}{2}$

62) 1

63) ∞

64) 0

65) 0

66) $e^{1/15}$

67) $-\frac{4}{5}$