Math 3B
Fall 2010
Sec 5.4

5.4/Second Fundamental Thm & Method of Substitution

5.4/10 $\int_1^4 \frac{s^4-8}{s^2}\,ds = \int_1^4 \left(\frac{s^4}{s^2} - \frac{8}{s^2}\right) ds$

$= \int_1^1 (s^2 - 8s^{-2})\,ds$

$= \frac{s^3}{3} - \frac{8s^{-1}}{-1}$

$= \left(\frac{s^3}{3} + \frac{8}{s}\right)\Big]_1^4 = \left(\frac{4^3}{3} + \frac{8}{4}\right) - \left(\frac{1^3}{3} + \frac{8}{1}\right)$

$= \frac{64}{3} + 2 - \frac{1}{3} - 8$

$= \frac{63}{3} - 6$

$= 21 - 6 = \boxed{15}$

16 $\int \sqrt[3]{2x-4}\,dx = \int (2x-4)^{1/3}\,dx$ let $u = 2x-4$, $du = 2\,dx$

$= \frac{1}{2}\int 2\,(2x-4)^{1/3}\,dx$

$= \frac{1}{2}\int (2x-4)^{1/3}\,2\,dx$

$= \frac{1}{2}\int u^{1/3}\,du = \frac{1}{2}\left(\frac{u^{4/3}}{\frac{4}{3}} + C\right)$

$\boxed{\frac{3}{8}(2x-4)^{4/3} + C}$ $= \frac{3}{8}u^{4/3} + C$

$= \frac{3}{8}\sqrt[3]{(2x-4)^4} + C$

5.4/ 20 $\int \cos(\pi v - \sqrt{7})\,dv$

let $u = \pi v - \sqrt{7}$
$du = \pi\,dv$

$= \frac{1}{\pi}\int \pi \cos(\pi v - \sqrt{7})\,dv$

$= \frac{1}{\pi}\int \cos(\pi v - \sqrt{7})\,\pi\,dv$

$= \frac{1}{\pi}\int \cos(u)\,du \quad = \frac{1}{\pi}\sin(u) + C$

$= \boxed{\frac{1}{\pi}\sin(\pi v - \sqrt{7}) + C}$

28 $\int \frac{z\cos(\sqrt[3]{z^2+3})}{(\sqrt[3]{z^2+3})^2}\,dz$

let $u = \sqrt[3]{z^2+3}$
$u = (z^2+3)^{1/3}$
$du = \frac{1}{3}(z^2+3)^{-2/3} \cdot 2z$

$= \frac{2z}{3[(z^2+3)^{\frac{1}{3}}]^2}$

$= \frac{2z}{3(\sqrt[3]{z^2+3})^2}$

$= \frac{3}{2}\int \frac{2z\cos(\sqrt[3]{z^2+3})}{3(\sqrt[3]{z^2+3})^2}\,dz$

$= \frac{3}{2}\int \cos(u)\,du$

$= \frac{3}{2}\sin(u) + C$

$= \boxed{\frac{3}{2}\sin(\sqrt[3]{z^2+3}) + C}$

5.4/#

32 $\int x^6 \sin(3x^7+9)\sqrt[3]{\cos(3x^7+9)}\,dx$

let $u = \cos(3x^7+9)$

$du = -\sin(3x^7+9)\cdot 21x^6\,dx$

$= -21x^6 \sin(3x^7+9)\,dx$

$= \frac{-1}{21}\int -21x^6 \sin(3x^7+9)\sqrt[3]{\cos(3x^7+9)}\,dx$

$= \frac{-1}{21}\int \sqrt[3]{u}\,du$

$= \frac{-1}{21}\int u^{1/3}\,du \quad = \frac{-1}{\cancel{21}_7}\cdot\frac{\cancel{3}}{4}u^{4/3} + C$

$= \frac{-u^{4/3}}{28} + C$

$= \boxed{\frac{-(\cos(3x^7+9))^{4/3}}{28} + C}$

5.4/ 38 $\int_2^{10} \frac{1}{y+4}\,dy$

let $u = y+4$
$du = 1dy$
$du = dy$

$a=2$ $y=2$ $u=y+4$
$=2+4=6=a_u$

a_u, b_u
new limits
of integration
for u.

$a=10$ $y=10$, $u=y+4$
$=10+4=14=b_u$

so you don't have to back substitute to calculate

$$\int_2^{10} \frac{1}{y+4}\,dy = \int_6^{14} \frac{du}{u} = \ln|u|\Big|_6^{14}$$

$= \ln 14 - \ln 6$

new limits of integration

$= \ln\left(\frac{14}{6}\right)$

$= \boxed{\ln\left(\frac{7}{3}\right)}$

5.5 4 Find the average of the function on the given interval.

$$f(x) = \frac{x^2}{\sqrt{x^3+16}} \quad \text{on } [0,2] = [a,b]$$

$$\frac{1}{b-a} = \frac{1}{2-0} = \frac{1}{2}$$

Average Value on $[0,2]$ $= \frac{1}{b-a}\int_a^b f(x)\,dx$

$$= \frac{1}{2}\int_0^2 \frac{x^2}{\sqrt{x^3+16}}\,dx$$

let $u = x^3+16$

$du = 3x^2\,dx$

$$= \frac{1}{2}\cdot\frac{1}{3}\int_0^2 \frac{3x^2\,dx}{\sqrt{x^3+16}}$$

$u = x^3+16$

$x=0 \quad u = 0^3+16 = 16$

$x=2 \quad u = 2^3+16 = 24$

$$= \frac{1}{6}\int_{16}^{24}\frac{du}{\sqrt{u}} = \frac{1}{6}\int_{16}^{24} u^{-1/2}\,du$$

$$= \frac{1}{6}\cdot\frac{2}{1}\cdot u^{1/2}\Big|_{16}^{24}$$

$$= \frac{1}{3}\left(\sqrt{24}-\sqrt{16}\right)$$

$$= \frac{1}{3}\left(2\sqrt{6}-4\right) = \boxed{\frac{2}{3}\left(\sqrt{6}-2\right)}$$

5.5 / 22) Find all values of c that satisfy the Mean Value Theorem for Integrals on the given interval.

$$g(y) = \cos(2y) \quad \text{on } [0,\pi]$$

Find all $c \in [0,\pi]$ where

$$g(c) = \frac{1}{\pi - 0}\int_0^{\pi} \cos(2y)\,dy$$

$$= \frac{1}{\pi}\int_0^{\pi} \cos(2y)\,dy$$

let $u = 2y$

$du = 2dy$

$y=0 \quad u = 2(0) = 0$

$y=\pi \quad u = 2(\pi) = 2\pi$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{1}{2}\cdot 2\cos(2y)\,dy$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \cos(u)\,du$$

$$= \frac{1}{2\pi}\sin u \Big|_0^{2\pi} = \frac{1}{2\pi}\left(\sin 2\pi - \sin(0)\right)$$

$$= \frac{0}{2\pi} = 0$$

where does.

$g(c) = 0$

$\cos(2c) = 0 \quad \text{in } [0,\pi]$

$2c = \frac{\pi}{2} + \frac{2\pi\cdot k}{2} \qquad k=0,1$

$c = \frac{\pi}{4} + \frac{2k\pi}{4}$

$k=0 \quad c = \pi/4 \qquad k=1 \quad c = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$