Name___________________________________

Identrify the critical points and find the maximum and minimum value on the given interval I.

1) $f(x) = x^2 + 18x + 81$; $I = [-18, 0]$ 1) _______________

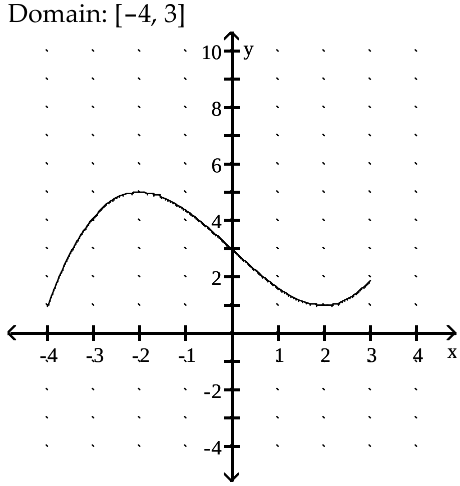
2) $f(r) = \dfrac{1}{r^2 + 2}$; $I = [-1, 5]$ 2) _______________

3) $r(\theta) = 2\cos\theta$; $I = \left[-\dfrac{\pi}{4}, \dfrac{\pi}{3}\right]$ 3) _______________

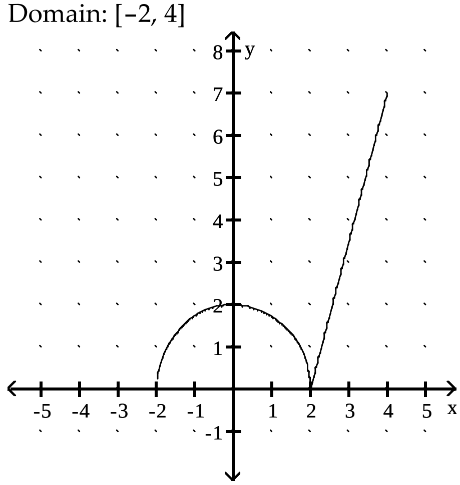
4) $f(x) = x^3 - 12x + 4$; $I = (-3, 5)$ 4) _______________

Find all critical points and find the minimum and maximum value of the function on the given domain.

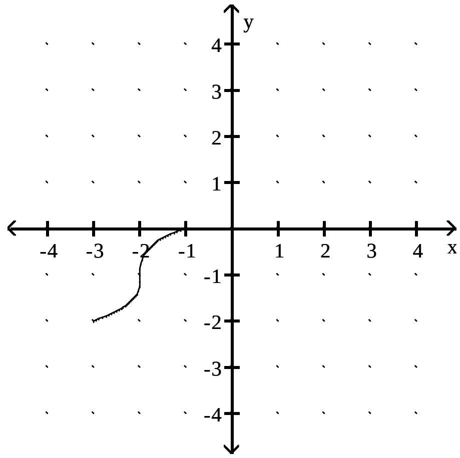
5) Domain: $[-4, 3]$ 5) _______________



6) Domain: $[-2, 4]$ 6) _______________



7) Domain: $[-3, -1]$ 7) ________________



Use the Concavity Theorem to determine where the given function is concave up and where it is concave down. Also find all inflection points.

8) $q(x) = 6x^3 + 2x + 3$ 8) ________________

9) $G(x) = \frac{1}{4}x^4 - x^3 + 11$ 9) ________________

10) $f(x) = x^3 + 3x^2 - x - 24$ 10) ________________

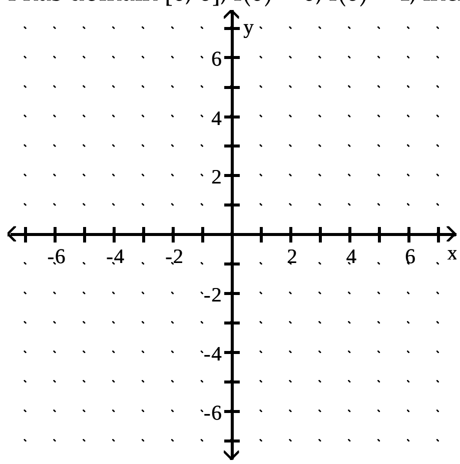
Use the Montonicity Theorem to find where the function is increasing and where it is decreasing.

11) $h(z) = 27z - z^3$ 11) ________________

12) $h(t) = \cos t, 0 \le t \le 2\pi$ 12) ________________

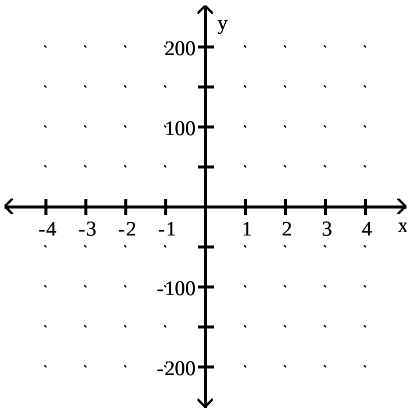
Sketch the graph of a continuous function f on the given domain that satisfies all conditions.

13) f has domain $[0, 6]$; $f(0) = 0$; $f(6) = 4$; increasing and concave down on $(0, 6)$ 13) ________________

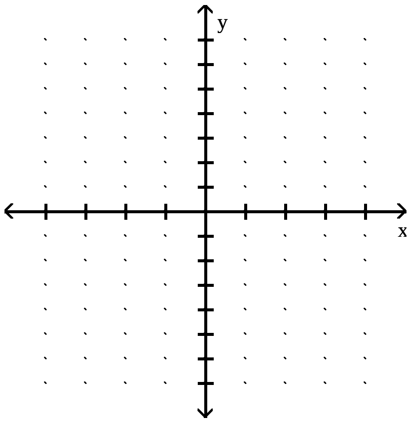


Determine where the graph of the function is increasing, decreasing, concave up, concave down. Then sketch the graph.

14) $g(x) = 3x^4 - 12x^3$ 14) ______________



15) $F(x) = 2x^3 + 9x^2 + 12x$ 15) ______________



Solve the problem.

16) Translate into the language of a derivative of the number of businesses with respect to time. N is the number of businesses in the downtown district after time t. The number of businesses downtown is decreasing at a slower and slower rate. 16) _______

A) $\frac{dN}{dt} > 0,\ \frac{d^2N}{dt^2} < 0$

B) $\frac{dN}{dt} < 0,\ \frac{d^2N}{dt^2} > 0$

C) $\frac{dN}{dt} < 0,\ \frac{d^2N}{dt^2} < 0$

D) $\frac{dN}{dt} < 0,\ \frac{d^2N}{dt^2} = k$, k is a constant

17) Translate into the language of a derivative of distance with respect to time. s is the position of the car at time t. The speed of the car is porportional to the distance it has traveled. 17) _______

A) $\frac{ds}{dt} = k$, k is a constant

B) $\frac{ds}{dt} = ks$, k is a constant

C) $\frac{d^2s}{dt^2} = ks$, k is a constant

D) $\frac{ds}{dt} = kt$, k is a constant

The first derivative f ' is given. Find all values of x that make the function a local minimum and a local maximum.

18) $f'(x) = (x - 6)^2(x + 7)$ 18) _______________

19) $f'(x) = (x + 4)(x + 8)$ 19) _______________

Identify the critical points. Then use the test of your choice to decide which critical points give a local maximum value and which give a local minimum value. Give these values.

20) $g(x) = \dfrac{x}{x^2 + 1}$ 20) _______________

Find, if possible, the (global) maximum and minimum values of the given function on the indicated interval.

21) $h(t) = \cos\left(t - \dfrac{\pi}{3}\right)$ on $\left[0, \dfrac{7\pi}{4}\right]$ 21) _______________

22) $g(x) = -x^2 + 5x - 6$ on $[2, 3]$ 22) _______________

Solve the problem.

23) If the price charged for a candy bar is p(x) cents, then x thousand candy bars will be sold in a certain city, where $p(x) = 62 - \dfrac{x}{12}$. How many candy bars must be sold to maximize revenue? 23) _______________

24) Suppose $c(x) = x^3 - 22x^2 + 20{,}000x$ is the cost of manufacturing x items. Find a production level that will minimize the average cost of making x items. 24) _______________

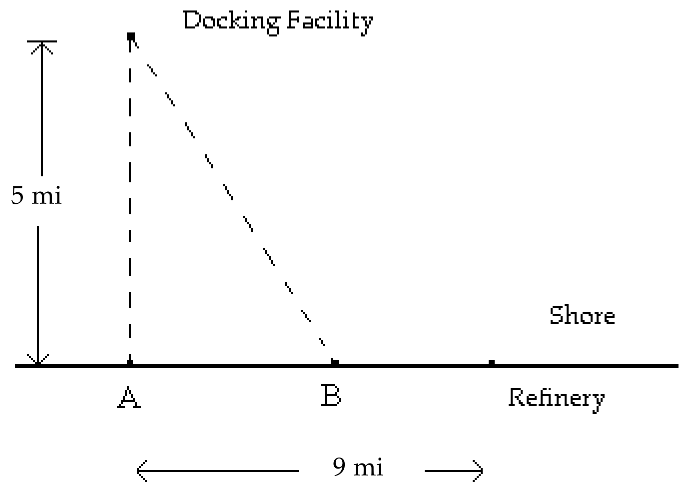
25) A baseball team is trying to determine what price to charge for tickets. At a price of $10 per ticket, it averages 50,000 people per game. For every increase of $1, it loses 5,000 people. Every person at the game spends an average of $5 on concessions. What price per ticket should be charged in order to maximize revenue? 25) _______________

26) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:
$R(x) = 40x - 0.5x^2$
$C(x) = 9x + 8$. 26) _______________

27) A company is constructing an open–top, square–based, rectangular metal tank that will have a volume of 48 cubic feet. What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary. 27) _______________

28) The velocity of a particle, in feet per second, is given by $v = t^2 - 8t + 2$, where t is the time (in seconds) for which it has traveled. Find the time at which the velocity is at a minimum. 28) _______________

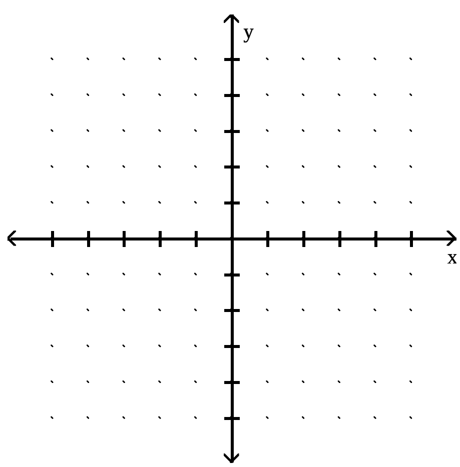
29) Supertankers off–load oil at a docking facility shore point 5 miles offshore. The nearest refinery is 9 miles east of the docking facility. A pipeline must be constructed connecting the docking facility with the refinery. The pipeline costs $300,000 per mile if constructed underwater and $200,000 per mile if over land. 29) _______________



Locate point B to minimize the cost of construction.

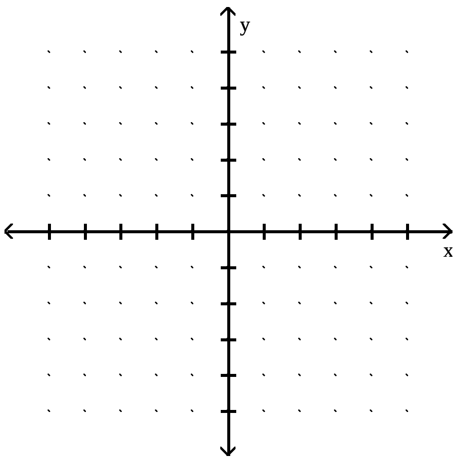
Sketch a graph of a function f that has the given properties.

30) (a) Defined for all real numbers 30) _______________
(b) Increasing for $-3 < x < 3$
(c) Decreasing for $-\infty < x < -3$ and $3 < x < \infty$
(d) Concave downward for $0 < x < \infty$
(e) Concave upward for $-\infty < x < 0$
(f) $f'(-3) = f'(3) = 0$
(g) Inflection point at (0, 0)



31) (a) Defined for all real numbers
(b) Increasing for $-3 < x < -1$ and $2 < x < \infty$
(c) Decreasing for $-\infty < x < -3$ and $-1 < x < 2$
(d) Concave upward for $-\infty < x < -2$ and $1 < x < \infty$
(e) Concave downward for $-2 < x < 1$
(f) $f'(-3) = f'(-1) = f'(2) = 0$
(g) Inflection point at $(-2, 0)$ and $(1, 1)$

31) ______________



L'Hopital's rule does not help with the given limit. Find the limit some other way.

32) $\lim\limits_{x \to \infty} \dfrac{\sqrt{36x + 1}}{\sqrt{x + 9}}$

32) ______________

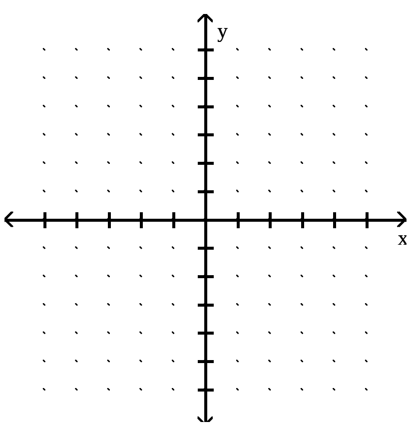
33) $\lim\limits_{x \to 0^+} \dfrac{1}{\cot x \ \sin x}$

33) ______________

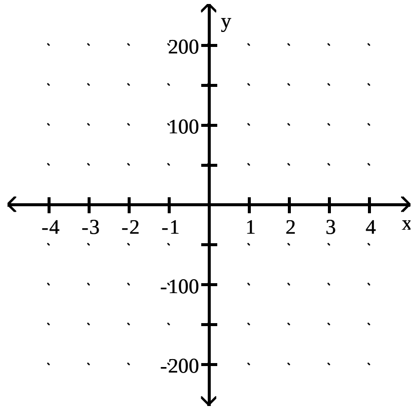
Make an analysis using calculus and sketch the graph.

34) $g(x) = \dfrac{6x}{x^2 + 1}$

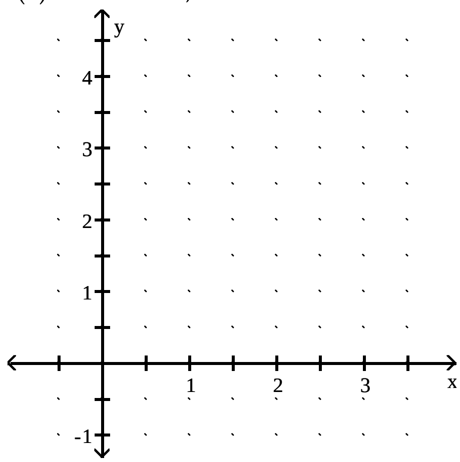
34) ______________



35) $g(x) = 3x^4 - 12x^3$ 35) _______________

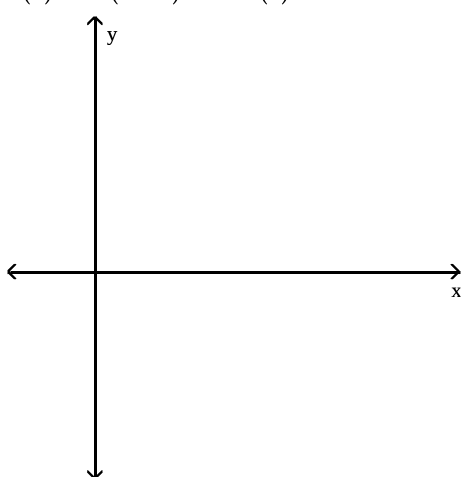


36) $f(x) = x + \cos 2x,\ 0 \le x \le \pi$ 36) _______________



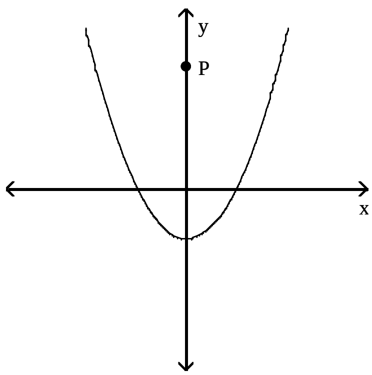
Sketch a possible graph of f(x) using f'(x).

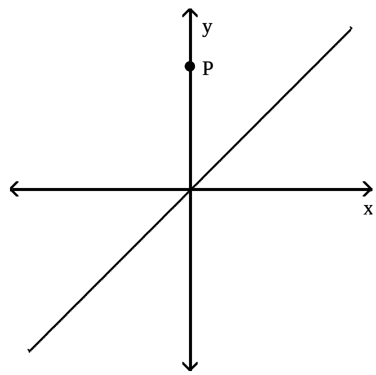
37) $f'(x) = x^2(2 - x)$ and $f(0) = 0$ 37) _______________



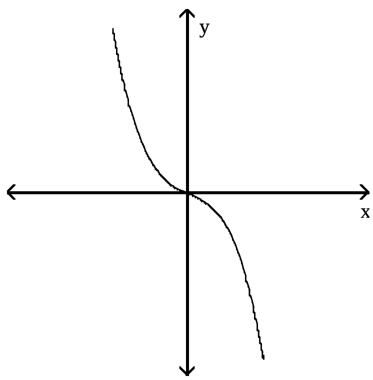
The graphs of the first and second derivatives of a function y = f(x) are given. Select a possible graph of f that passes through the point P. (NOTE: Vertical scales may vary from graph to graph.)

38) f' f'' 38) _______

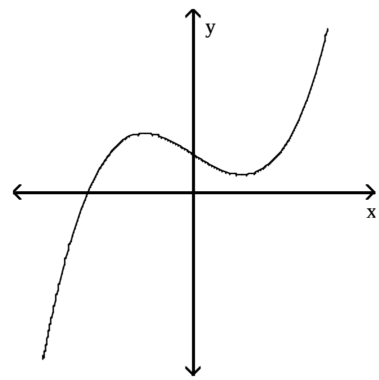




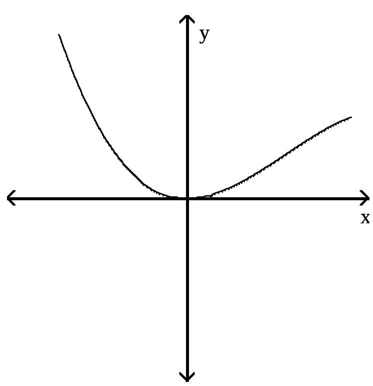
A)



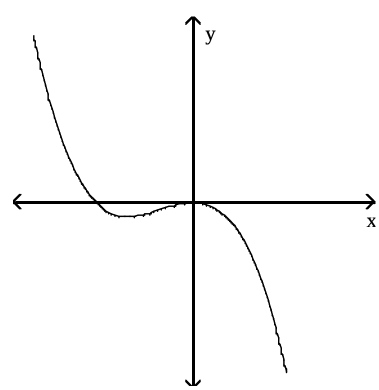
B)



C)



D)



Provide an appropriate response.

39) A marathoner ran the 26.2 mile New York City Marathon in 2.8 hrs. Did the runner ever exceed a speed of 9 miles per hour? 39) _______________

40) A trucker handed in a ticket at a toll booth showing that in 2 hours he had covered 215 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why? 40) _______________

Decide whether the Mean Value Theorem applies to the given function on the given interval.

41) $h(t) = \sqrt{t(1 - t)}$; $[-1, 5]$ 41) _______________

42) $g(x) = x^{3/4}$; [0, 1] 42) ______________

Use the Mean Value Theorem and find all possible values of c on the given interval.

43) $f(x) = x + \frac{54}{x}$; [6, 9] 43) ______________

Approximate the values of x that gives the maximum and minimum values of the function on the indicated intervals.

44) $f(x) = x^4 - x^3 - x^2 - x$; [0, 2] 44) ______________

Find the indicated root of the given equation by using Newton's method.

45) $x^3 - x - 1 = 0$ (between 1 and 2) 45) ______________

46) $2x^4 - 3x^2 - 7x + 1 = 0$ (between 0 and 1) 46) ______________

Evaluate the indefinite integral.

47) $\int (7x^3 + 7x + 8)\,dx$ 47) ______________

48) $\int \left(\frac{\sqrt{y}}{4} + \frac{3}{\sqrt{y}}\right)dy$ 48) ______________

49) $\int \left(2t^2 + \frac{t}{3}\right)dt$ 49) ______________

Find the general antiderivative F(x) + C for the function.

50) $f(x) = \frac{5}{4}x^{3/4}$ 50) ______________

51) $f(x) = 8x + 6\pi^5$ 51) ______________

52) $f(x) = \frac{8x^7 + 5x^5}{x^4}$ 52) ______________

53) $f(x) = x^{-3} + \frac{1}{3\sqrt{x}}$ 53) ______________

54) $f(x) = 2\sqrt[8]{x} - 3$ 54) ______________

55) $f(x) = 5x - 5$ 55) ______________

Find the particular solution that satisfies the given condition.

56) $\frac{dy}{dx} = x - 6$; curve passes through (2, 5) 56) ______________

57) $\frac{dy}{dx} = 2x^{-3/4}$; curve passes through (1, 3)

57) ______________

58) $\frac{dy}{dx} = \frac{x}{y}$; y = 1 at x = 0

58) ______________

59) $\frac{du}{dt} = u^3(t - 2t^3)$; u = 3 at x = 0

59) ______________

60) $\frac{dy}{dx} = \frac{1}{x^3} + x, x > 0$; curve passes through (2, 0)

60) ______________

Use L'Hôpital's rule to find the limit.

61) $\lim_{x \to 0} \frac{\cos 3x - 1}{x^2}$

61) ______________

62) $\lim_{x \to 0} \frac{\sin (5x)}{\sin x}$

62) ______________

63) $\lim_{x \to 0} \frac{\sin x^9}{x}$

63) ______________

64) $\lim_{x \to \frac{\pi}{3}} \frac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}}$

64) ______________

65) $\lim_{x \to 9} \frac{x^2 - 81}{x - 9}$

65) ______________

66) $\lim_{x \to 0} \frac{e^x - 1}{\sin 7x}$

66) ______________

67) $\lim_{x \to 0} \frac{5^x - 1}{7^x - 1}$

67) ______________

68) $\lim_{x \to \infty} \frac{x^2 + 6x + 11}{x^3 - 3x^2 + 4}$

68) ______________

69) $\lim_{x \to \infty} x \sin \frac{5}{x}$

69) ______________

70) $\lim_{x \to \infty} \left(1 + \frac{3}{x^4}\right)^x$ 70) ______________

71) $\lim_{x \to \infty} (\sqrt{x^2 + 7x} - x)$ 71) ______________

72) $\lim_{x \to \infty} \frac{15x^2 - 8x - 7}{10x^2 - 7x + 8}$ 72) ______________

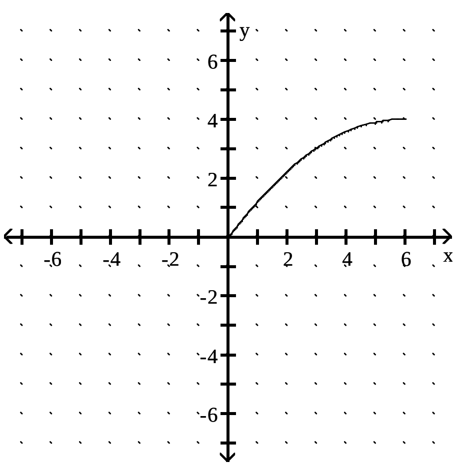
73) $\lim_{x \to 0^+} (x^2 \ln x)$ 73) ______________

74) $\lim_{x \to 0} 10x \csc x$ 74) ______________

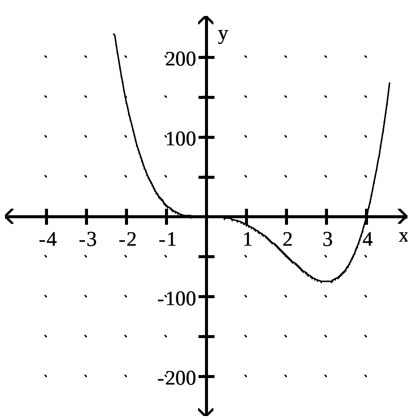
75) $\lim_{x \to \infty} \frac{3x^{576}}{e^x}$ 75) ______________

Answer Key
Testname: MATH3A_CH4_APPLICATIONS_PRACTICE

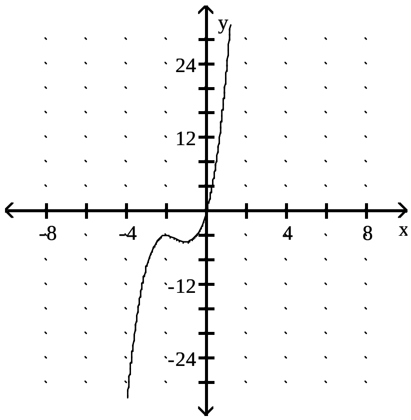
1) Critical points: $-18, -9, 0$; maximum value 81; minimum value 0
2) Critical points: $-1, 0, 5$; maximum value $\frac{1}{2}$; minimum value $\frac{1}{27}$
3) Critical points: $-\frac{\pi}{4}, 0, \frac{\pi}{3}$; maximum value 2; minimum value 1
4) Critical points: $-2, 2$; no maximum value; minimum value -12
5) Critical points: $-4, -2, 2, 3$; maximum value: 5; minimum value: 1
6) Critical points: $-2, 0, 2, 4$; maximum value: 7; minimum value: 0
7) Critical points: $-3, -2, -1$; maximum value: 0; minimum value: -2
8) Concave up on $(0, \infty)$, concave down on $(-\infty, 0)$; inflection point $(0, 3)$
9) Concave up on $(-\infty, 0) \cup (2, \infty)$, concave down on $(0, 2)$; inflection points $(0, 11)$ and $(2, 7)$
10) Concave up on $(-1, \infty)$, concave down on $(-\infty, -1)$; inflection point $(-1, -21)$
11) Increasing on $[-3, 3]$, decreasing on $(-\infty, -3] \cup [3, \infty)$
12) Increasing on $[\pi, 2\pi]$, decreasing on $[0, \pi]$
13)



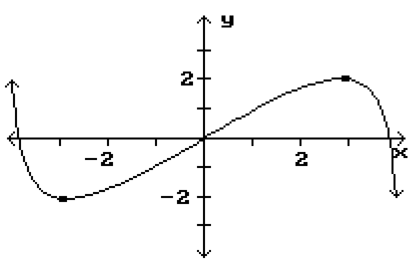
14)



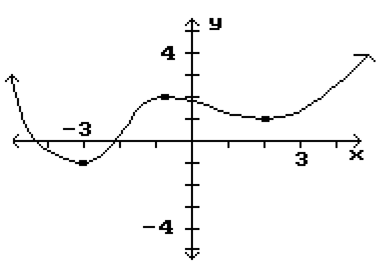
15)



16) B
17) B
18) Local minimum at $x = -7$
19) Local minimum at $x = -4$; local maximum at $x = -8$
20) Critical points: -1, 1; local maximum $f(1) = \frac{1}{2}$; local minimum $f(-1) = -\frac{1}{2}$
21) Maximum value $h\left(\frac{\pi}{3}\right) = 1$; minimum value $h\left(\frac{4\pi}{3}\right) = -1$
22) Maximum value $g\left(\frac{5}{2}\right) = \frac{1}{4}$; minimum value $g(3) = g(2) = 0$
23) 372 thousand candy bars
24) 11 items
25) $7.50
26) 31 units
27) 4.6 ft by 4.6 ft. by 2.3 ft
28) 4 sec
29) Point B is 4.47 miles from Point A.
30)

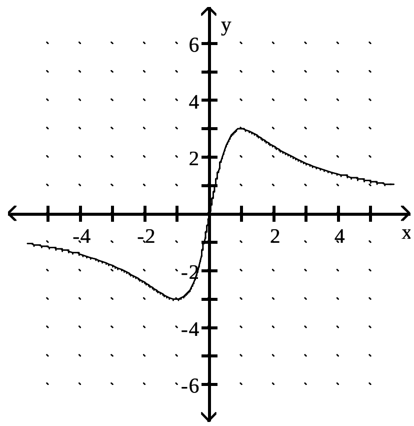


31)



32) 6
33) 1

34)



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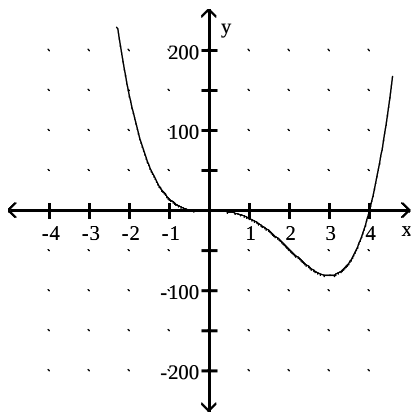

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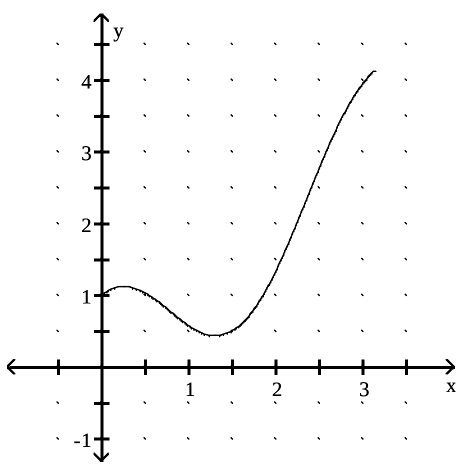

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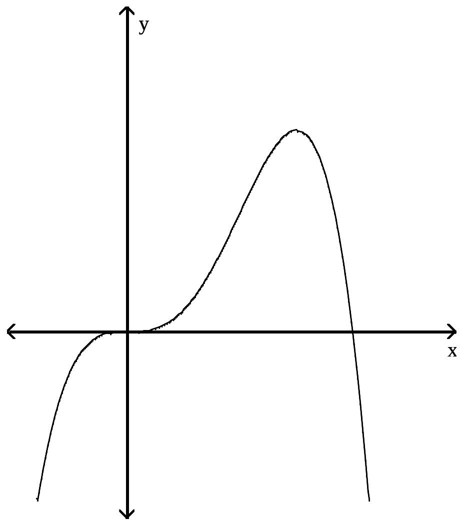

38) B

39) Yes, the Mean Value Theorem implies that the runner attained a speed of 9.4 mph, which was her average speed throughout the marathon.

40) As the trucker's average speed was 108 mph, the Mean Value Theorem implies that the trucker must have been going that speed at least once during the trip.

41) No

42) Yes

43) $c = 3\sqrt{6} \approx 7.35$

44) Minimum $f(1.28858) \approx -2.33157$; maximum $f(2) = 2$

45) 1.3247180

46) 0.1351270

47) $\frac{7}{4}x^4 + \frac{7}{2}x^2 + 8x + C$

48) $\frac{1}{6}y^{3/2} + 6\sqrt{y} + C$

49) $\frac{2}{3}t^3 + \frac{t^2}{6} + C$

50) $\frac{5}{7}x^{7/4} + C$

51) $4x^2 + 6\pi^5 x + C$

52) $2x^4 + \frac{5}{2}x^2 + C$

53) $-\frac{1}{2x^2} + \frac{2}{3}x^{1/2} + C$

54) $\frac{16}{9}x^{9/8} - 3x + C$

55) $\frac{5}{2}x^2 - 5x + C$

56) $y = \frac{x^2}{2} - 6x + 15$

57) $y = 8x^{1/4} - 5$
58) $y = \sqrt{x^2 + 1}$
59) $u = \dfrac{1}{\sqrt{t^4 - t^2 + \frac{1}{9}}}$
60) $y = -\dfrac{1}{2x^2} + \dfrac{x^2}{2} - \dfrac{15}{8}$
61) $-\dfrac{9}{2}$
62) 5
63) 0
64) $-\dfrac{\sqrt{3}}{2}$
65) 18
66) $\dfrac{1}{7}$
67) $\dfrac{\ln 5}{\ln 7}$
68) 0
69) 5
70) 1
71) $\dfrac{7}{2}$
72) $\dfrac{3}{2}$
73) 0
74) 10
75) 0