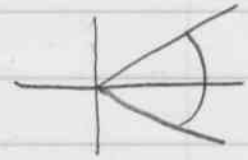
Calc I

4.1 Maxima/Minima

(17) $r(\theta) = \sin\theta \;;\; I = \left[-\frac{\pi}{4}, \frac{\pi}{6}\right]$



$r'(\theta) = \cos\theta \neq 0$ on $-\frac{\pi}{4}, \frac{\pi}{6}$

Critical pts = endpts $\theta = -\frac{\pi}{4}$ $\quad \theta = \frac{\pi}{6}$

$\sin(-\frac{\pi}{4}) = -\sin(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$ $\Leftarrow$ min value $y = -\frac{\sqrt{2}}{2}$ at $x = -\frac{\pi}{4}$

$\sin(\frac{\pi}{6}) = \frac{1}{2}$ $\Leftarrow$ max value $y = \frac{1}{2}$ at $x = \frac{\pi}{6}$

4.1

19 $a(x) = |x-1|$; $I = [0,3]$

$$a(x) = |x-1| = \begin{cases} -(x-1) & 0 \le x \le 1 \quad a'(x) = -1 \\ x-1 & 1 \le x \le 3 \quad a'(x) = 1 \end{cases}$$

Critical pts @ $x=0,\ x=1,\ x=3$

$a(0) = |0-1| = |-1| = 1$

$a(1) = |1-1| = 0.$ ⟵ min value $y=0$ at $x=1$

$a(3) = |3-1| = |2| = 2$ ⟵ max value $y=2$ at $x=3$.

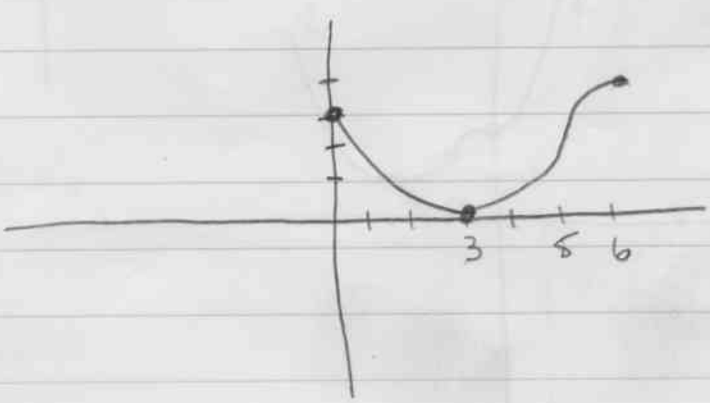
4.2

31 Sketch graph of continuous function on $[0,6]$ that satisfies all the stated conditions

$f(0)=3 \quad f(3)=0 \quad f(6)=4$

	(0,3)	(3,5)	(5,6)
$f'(x)$	neg decrease	pos increase	pos increase
$f''(x)$	pos concave up	pose concave up	neg concave down
	\ + U =	/ + U =	/ + ∩ =

inflection



(21) Sketch the graph using f' and f''

$g(x) = 3x^4 - 4x^3 + 2$ → no I so $(-\infty, \infty)$

$g'(x) = 12x^3 - 12x^2 = 0$

when $12x^2(x-1) = 0$

$x=0$ and $x=1$ crit. pts.

$f(0) = 2$

$f\left(\frac{2}{3}\right) = \frac{16}{27} - \frac{32}{27} + \frac{54}{27} = \frac{38}{27}$

$f(1) = 3 - 4 + 2 = 1$

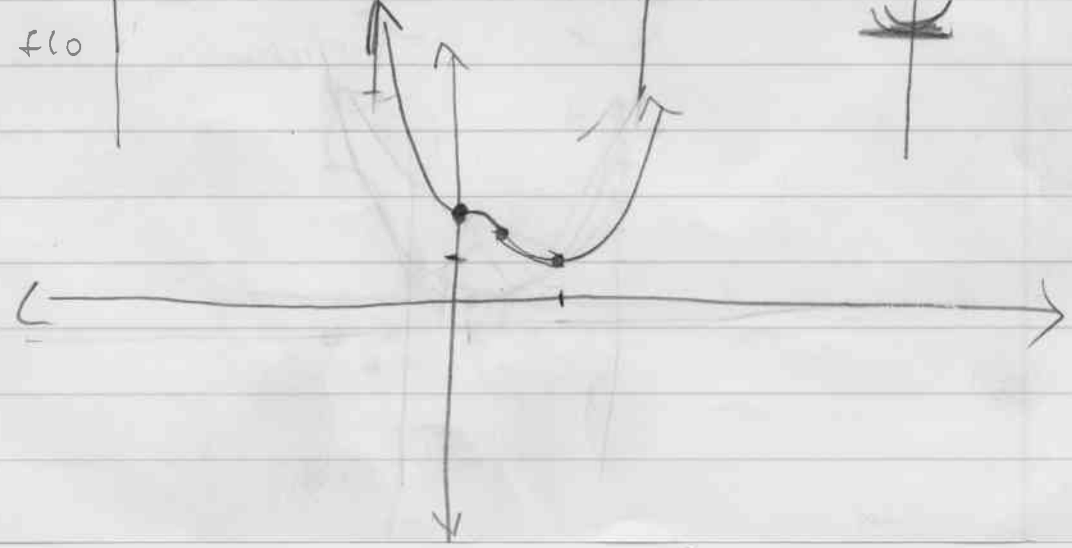
$g''(x) = 36x^2 - 24x = 0$

when $12x(3x-2) = 0$

$\boxed{x=0 \quad x=\frac{2}{3}}$ crit pts.

$0, \frac{2}{3}, 1$

test pt.	$(-\infty, 0)$	$(0, \frac{2}{3})$	$(\frac{2}{3}, 1)$	$(1, \infty)$
testpt	$x=-1$	$x=\frac{1}{3}$	$x=5/6$	$x=2$
$g'(x) = 12x^2(x-1)$	(+)(−) = neg decrease	(+)(−) = neg decrease	(+)(−) = neg decrease	(+)(+) = pos increase
$g''(x) = 12x(3x-2)$	(−)(−) = + concave up.	(+)(−) = neg concave down	(+)(+) = + concave up	(+)(+) = (+) concave up
	\ + U = L	\ + ∩ = L	\ + U = L	/ + U = J
f(0)				



Calc

4.1 / Maxima / Minima

(5) Identify the critical points and identify the max value and min value on the given interval

$f(x) = x^2 + 4x + 4 \quad I = [-4, 0]$

$f'(x) = 2x + 4 \quad = 0$

when $2x = -4$

$x = -4/2$

$x = -2$

Critical pts: $x = -4, -2, 0$

$f(-4) = (-4)^2 + 4(-4) + 4$

$= 16 - 16 + 4$

$= 4$ ✓

$f(-2) = (-2)^2 + 4(-2) + 4$

$= 4 - 8 + 4$

$= 0$

$f(0) = (0)^2 + 4(0) + 4$

$= 4$ ✓

max values 4

← min values 0

(16) $f(x) = \frac{x}{1+x^2}$; $I = [-1, 4]$

$f'(x) = \frac{1(1+x^2) - x(2x)}{(1+x^2)^2} = 0$

when $\boxed{\begin{array}{c} f'(x)=0 \\ \text{num} = 0 \end{array}}$ $\boxed{\begin{array}{c} f'(x) \not\exists \\ \text{den} = 0 \end{array}}$

$1+x^2 - 2x^2 = 0$

$1 - x^2 = 0$

$(1-x)(1+x) = 0$

$1-x=0$ OR $1+x=0$

$\boxed{x=1}$ OR $\boxed{x=-1}$

Critical pts: $x = -1, 1, 4$

$f(-1) = \frac{-1}{1+(-1)^2} = -\frac{1}{2}$ ← min value $y = -\frac{1}{2}$ at $x = -1$

$f(1) = \frac{1}{1+(1)^2} = \frac{1}{2}$ ← max value $y = \frac{1}{2}$ at $x = 1$ crit pt.

$f(4) = \frac{4}{1+(4)^2} = \frac{4}{17}$