4.5 #16

16. $w(z) = \dfrac{z^2+1}{z}$

$w'(z) = \dfrac{2z \cdot z - (z^2+1)\cdot 1}{z^2} = \dfrac{2z^2 - z^2 - 1}{z^2} = \dfrac{z^2-1}{z^2} = 0$

$w''(z) = \dfrac{2z \cdot z^2 - (z^2-1)\cdot 2z}{z^4}$

$= \dfrac{2z^3 - 2z^3 + 2z}{z^4} = \dfrac{2z}{z^4} = \dfrac{2}{z^3}$

when $(z^2-1) = (z-1)(z+1) = 0$

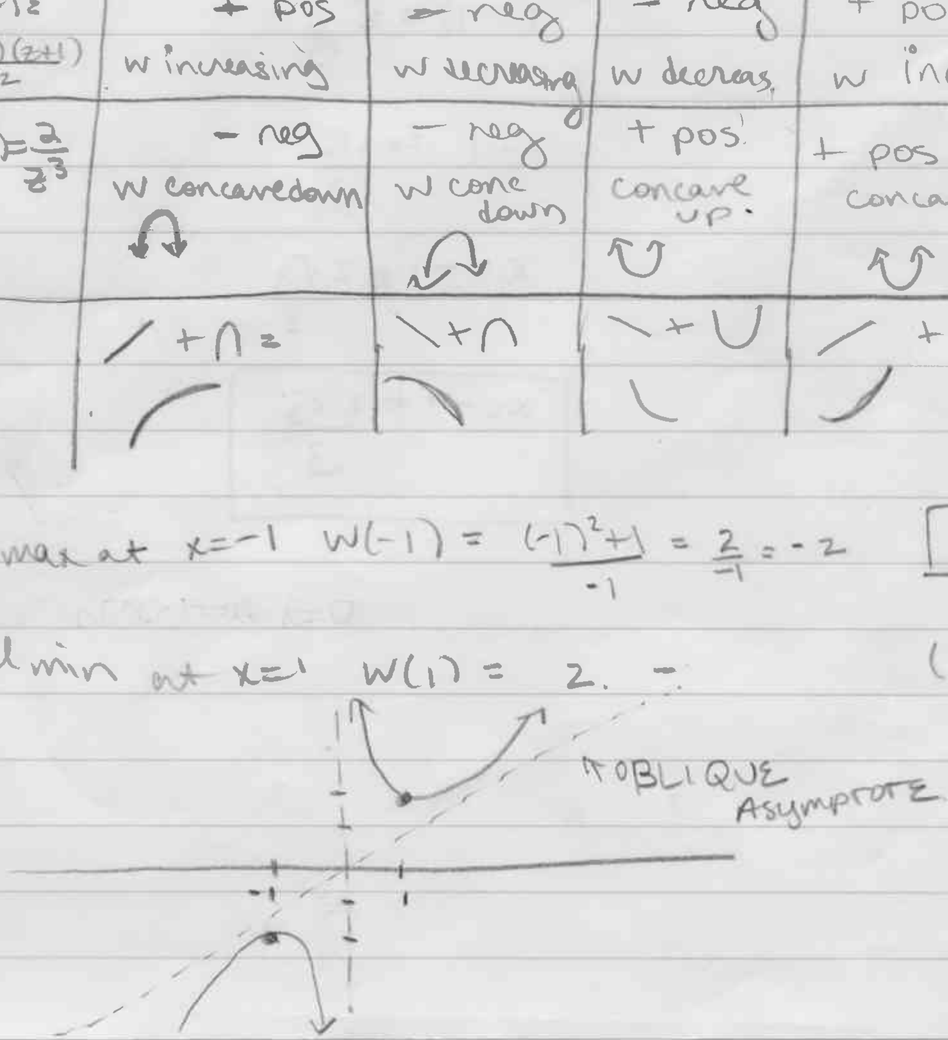
$z = 1,\ z = -1$

and $\nexists$ when $z = 0$

	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Test pt	-2	$-\frac{1}{2}$	$\frac{1}{2}$	2
$w'(z) = \dfrac{(z-1)(z+1)}{z^2}$	+ pos w increasing	− neg w decreasing	− neg w decreas.	+ pos w increasing
$w''(z) = \dfrac{2}{z^3}$	− neg w concavedown	− neg w conc down	+ pos concave up	+ pos concave up
	/ + ∩ =	\ + ∩	\ + ∪	/ + ∪

rel max at $x=-1$ $\quad w(-1) = \dfrac{(-1)^2+1}{-1} = \dfrac{2}{-1} = -2$ $\quad \boxed{(-1,-2)}$

rel min at $x=1$ $\quad w(1) = 2.$ $\quad (1, 2)$



4.6

(2) $g(x) = |x|$; $[-2, 2]$

Mean Value thm doesn't apply

(4) $g(x) = (x+1)^3$, $[-1, 1]$

$g'(x) = 3(x+1)^2$

$$\frac{g(b)-g(a)}{b-a} = \frac{g(1)-g(-1)}{1-(-1)} = \frac{8-0}{2} = 4$$

$$g'(x) = 3(x+1)^2 = 4$$

$$(x+1)^2 = \frac{4}{3}$$

$$(x+1) = \pm \frac{2}{\sqrt{3}}$$

$$x+1 \quad 2 \pm \frac{2\sqrt{3}}{3}$$

$$x = -1 \pm \frac{2\sqrt{3}}{3}$$

$$\boxed{x = -1 + \frac{2\sqrt{3}}{3}}$$

4.6 (MVT)

Decide whether the Mean Value Theorem applies to the given function on the given interval. If it does, find all possible values of c; if not state the reason.

(4) $g(x) = (x+1)^3$; on $[-1,1]$

$g(x)$ is continuous & differentiable on $[-1,1]$ so MVT applies.

$g'(x) = 3(x+1)^2$

$\dfrac{g(1)-g(-1)}{1-(-1)}$ = Avg. Value on $[-1,1]$

set Avg Val $= g'(x)$ & Find x

$\dfrac{2^3 - 0^3}{2} = 4$ = Avg. VAL on $[-1,1]$

$4 = 3(x+1)^2$

$4 = 3(x^2+2x+1)$

$4 = 3x^2+6x+3$

$0 = 3x^2+6x-1$

$x = \dfrac{-6 \pm \sqrt{36+12}}{12}$

$x = \dfrac{-6 \pm \sqrt{48}}{12}$ only

$\boxed{x = \dfrac{-6+\sqrt{49}}{12} \text{ is in the interval } [-1,1] \quad \text{let } c = x.}$

let x be the value of c. then $g'(c)$ = Avg value.

8.1 L'Hôpital's Rule 0/0

(2) Find the limit. Make sure L'Hôpital's Rule (LH) applies before using it.

$$\lim_{x \to \pi/2} \frac{\cos x}{\frac{1}{2}\pi - x} = \frac{0}{0}$$ indeterminate form so LH applies

$$\lim_{x \to \pi/2} \frac{\cos x}{\frac{1}{2}\pi - x} = \lim_{x \to \pi/2} \frac{-\sin x}{-1} = \frac{-1}{-1} = \boxed{1}$$

(4) $$\lim_{x \to 0} \frac{\tan^{-1} 3x}{\sin^{-1} x} = \frac{\tan^{-1} 0}{\sin^{-1} 0} = \frac{0}{0}$$ indeterminate LH applies.

$$\lim_{x \to 0} \frac{\tan^{-1} 3x}{\sin^{-1} x} = \lim_{x \to 0} \frac{\frac{3}{1-9x^2}}{\frac{1}{\sqrt{1-x^2}}}$$

$$= \frac{\frac{3}{1-0}}{\frac{1}{\sqrt{1-0}}} = \frac{3}{1} = \boxed{3}$$