

3.2

Use the Definition $f'(c) = \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$

to find the derivative.

(2) $f'(2)$ if $f(t) = (2t)^2$

$$f'(2) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(4+2h)^2 - 4^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{16 + 16h + 4h^2 - 16}{h}$$

$$= \lim_{h \rightarrow 0} \frac{16h + 4h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h \cdot (16 + 4h)}{h}$$

$$= \lim_{h \rightarrow 0} 16 + 4h$$

$$= 16 + 4(0)$$

$$= \boxed{16}$$

The Derivative

2.3

(20) $g(x) = \frac{1}{\sqrt{3x}}$

$$g'(x) = \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{3(x+h)}} - \frac{1}{\sqrt{3x}}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3x} - \sqrt{3(x+h)}}{\sqrt{3(x+h)} \cdot \sqrt{3x} \cdot h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3x} - \sqrt{3(x+h)}}{h \cdot (\sqrt{9x(x+h)})} \cdot \frac{(\sqrt{3x} + \sqrt{3(x+h)})}{(\sqrt{3x} + \sqrt{3(x+h)})}$$

$$= \lim_{h \rightarrow 0} \frac{3x - 3(x+h)}{h(\sqrt{9x(x+h)})(\sqrt{3x} + \sqrt{3(x+h)})}$$

$$= \lim_{h \rightarrow 0} \frac{-3h}{h(\sqrt{9x(x+h)})(\sqrt{3x} + \sqrt{3(x+h)})}$$

$$= \lim_{h \rightarrow 0} \frac{-3}{\sqrt{9x(x+h)} \cdot (\sqrt{3x} + \sqrt{3(x+h)})}$$

$$= \frac{-3}{\sqrt{9x(x+0)} \cdot (\sqrt{3x} + \sqrt{3(x+0)})}$$

$$= \frac{-3}{\sqrt{9x^2} (2\sqrt{3x})} = \frac{-3}{6x\sqrt{3x}}$$

$$= \frac{-1}{2x\sqrt{3x}} = \frac{-\sqrt{3x}}{6x^2}$$

The Derivative

3.2

(24) Use $f'(x) = \lim_{t \rightarrow x} \frac{f(t) - f(x)}{t - x}$ to find $f'(x)$

$$f(x) = x^3 + 5x$$

$$f(t) = t^3 + 5t$$

$$f'(x) = \lim_{t \rightarrow x} \frac{f(t) - f(x)}{t - x}$$

$$= \lim_{t \rightarrow x} \frac{(t^3 + 5t) - (x^3 + 5x)}{t - x}$$

$$= \lim_{t \rightarrow x} \frac{t^3 - x^3 + 5t - 5x}{t - x}$$

clean ()
and
rearrange
terms.

$$= \lim_{t \rightarrow x} \frac{(t-x)(t^2 + tx + x^2) + 5(t-x)}{t-x}$$

factor (t-x)
by grouping

$$= \lim_{t \rightarrow x} \frac{\cancel{(t-x)} [t^2 + tx + x^2 + 5]}{\cancel{(t-x)}}$$

$$= \lim_{t \rightarrow x} t^2 + tx + x^2 + 5$$

$$= (x)^2 + (x)x + x^2 + 5$$

$$= x^2 + x^2 + x^2 + 5$$

$$= \boxed{3x^2 + 5}$$