

Slope of Tangent Line  
Instantaneous Velocity

3.1

- (10) Find the slopes of the tangent lines to the curve  $y = x^3 - 3x$  at the points where  $x = -2, -1, 0, 1, 2$ .

Find "formula" for slope of tangent line.

use  $\lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h} = m$

Then substitute  $x$ -values for  $c$ .

$$\lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h} \quad \text{for } f(x) = x^3 - 3x$$

$$= \lim_{h \rightarrow 0} \frac{[(c+h)^3 - 3(c+h)] - [c^3 - 3c]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{c^3 + 3c^2h + 3ch^2 + h^3 - 3c - 3h - c^3 + 3c}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3c^2h + 3ch^2 + h^3 - 3h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h} \cdot (3c^2 + 3ch + h^2 - 3)}{\cancel{h}}$$

$$= \lim_{h \rightarrow 0} 3c^2 + 3ch + h^2 - 3$$

slope Formula!

$$= 3c^2 + 3c(0) + (0)^2 - 3 = \boxed{3c^2 - 3}$$

for  $x = -2$  slope =  $3(-2)^2 - 3 = \boxed{9}$

$x = -1$  slope =  $3(-1)^2 - 3 = \boxed{0}$  etc...

# Equation of Tangent line to Curve

(12) Find the equation of the tangent line to  $y = \frac{1}{x-1}$  at  $(0, -1)$

To find equation of a line we need a point  $(x_1, y_1)$  and a slope  $m$ .

We have the point  $(x_1, y_1) = (0, -1)$

First, Find slope  $m$  at  $x = 0$ .

$$m = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} \quad f(x) = \frac{1}{x-1}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{h-1} + 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{h-1} + \frac{h-1}{h-1}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1+h-1}{h-1}}{h}$$

$$= \frac{\frac{h}{h-1}}{h} = \frac{h}{h(h-1)}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h(h-1)}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h-1} = \frac{1}{0-1} = -1$$

slope  $m = -1$ .

(12) Continued

point-slope form for equation of tangent line is

$$y - y_1 = m(x - x_1)$$

$$(x_1, y_1) = (0, -1)$$
$$m = -1$$

$$y - (-1) = -1(x - 0)$$

$$y + 1 = -x$$

$$\boxed{y = -x - 1}$$

→ equation of tangent line to the curve

$$y = \frac{1}{x-1} \quad \text{at pt. } (0, -1)$$

18) A business is prospering in such a way that its total (accumulated) profit after  $t$  years is  $1000t^2$  dollars.

(a) How much did the business make during the third year (between  $t=2$ ,  $t=3$ )?

Money earned 3rd year =

$$\begin{aligned} & \text{Accumulated } t=3 - \text{Accumulated } t=2 \\ & = 1000(3)^2 - 1000(2)^2 \end{aligned}$$

$$= \$5,000$$

(b) What was the average rate of profit during the first half of the third year (between  $t=2$  and  $t=2.5$ )?

$$\text{Average rate} = \frac{1000(2.5)^2 - 1000(2)^2}{2.5 - 2} \quad \$/\text{yr.}$$

$$= 2250 / .5 = \$4500/\text{yr.}$$

rate!

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(18) continued ...  $\rightarrow$  \$4000

(c) What was its instantaneous rate of profit at  $t=2$ .

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{1000(2+h)^2 - 1000(2)^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{1000(4+4h+h^2) - 4000}{h} \\ &= \lim_{h \rightarrow 0} \frac{4000 + 4000h + 1000h^2 - 4000}{h} \\ &= \lim_{h \rightarrow 0} 4000 + 1000h = 4000 + 1000(0) \\ & \quad \boxed{\$4,000 / \text{yr}} \end{aligned}$$