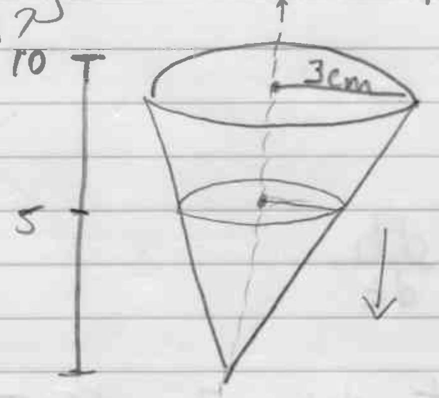
3.8 Related Rates

(4) A student is using a straw to drink from a conical paper cup, whose axis is vertical, at a rate of 3 cm^3/sec. If the height of the cup is 10cm and the diameter of its opening is 6 cm, how fast is the level of the liquid falling when the depth of the liquid is 5cm?



(1) Draw Picture
(2) Label facts
(3) Find question and identify what you need

Find $\frac{dh}{dt}$ when $h=5$

$\frac{dV}{dt} = 3$

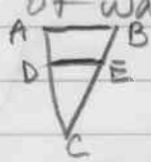
(4) Implicit diff!

Volume Cone $= V = \frac{1}{3}\pi r^2 \cdot h$ ← make into 2 variables V and h for the facts we have and want to find

(if we needed to find dr/dt, then we'd write formula in V and r instead.)

Height of the water in cone is always parallel to other heights, the vertex of the cone, axis, and side form a triangle

And any height of water will form similar triangles



$\triangle ABC \sim \triangle DEC$

with corresponding proportional sides.

$\frac{r}{h} = \frac{3}{10}$

$r = 3/10\, h$

$V = \frac{1}{3}\pi r^2 h$ $\qquad \frac{r}{h} = \frac{3}{10}$

$r = \frac{3}{10}h$

$V = \frac{1}{3}\pi\left(\frac{3}{10}h\right)^2 \cdot h$

Implicit Differentiation

$V = \frac{3\pi h^3}{100}$

$\frac{dV}{dt} = \frac{3 \cdot 3\pi h^2}{100} \cdot \frac{dh}{dt}$

Now evaluate at $\frac{dV}{dt} = 3$, $h = 5$

Then solve for $\frac{dh}{dt}$

$3 = \frac{9\pi (5)^2}{100} \cdot \frac{dh}{dt}$

$3 = \frac{9\pi}{4} \cdot \frac{dh}{dt}$

$\frac{3 \cdot 4}{9\pi} = \frac{dh}{dt}$

$\frac{4}{3\pi} = \frac{dh}{dt}$

Answer: At $h=5$ height of water is falling $\frac{4}{3\pi}$ cm/sec

$\approx$ 0.42 cm/sec

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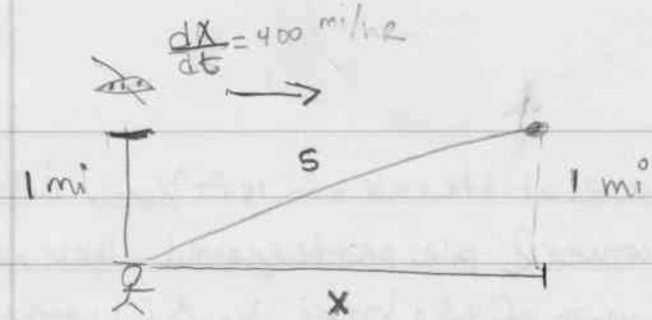


(3)

How fast is the airplane's distance from the observer increasing 45 sec later?

$45 \text{ sec} = \frac{3}{4} \text{ min} = \frac{3}{4} \text{ min} \cdot \frac{1 \text{ hR}}{60}$

$\boxed{t = \frac{1}{80} \text{ hR}}$

$\frac{dx}{dt} = 400$

$x^2 + 1^2 = s^2$ relationship between variables.

implicit differentiation to find

$\frac{ds}{dt}$ = rate of change of distance from observer to airplane

$2x \cdot \frac{dx}{dt} + 0 = 2s \cdot \frac{ds}{dt}$

$2x \frac{dx}{dt} = 2s \frac{ds}{dt}$

$\frac{x}{s} \frac{dx}{dt} = \frac{ds}{dt}$ at $t = \frac{1}{80}$ hR

$\frac{5}{\sqrt{26}} \cdot 400 = \frac{ds}{dt}$

X = distance airplane travelled @ 400 miles per hR

= rate · time

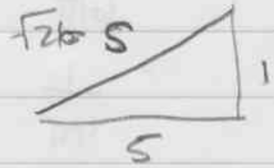
$= 400 \cdot \frac{1}{80}$

$= \frac{\cancel{400}}{\cancel{80}}$

X = 5 mi

$\frac{2000}{\sqrt{26}} = \frac{2000\sqrt{26}}{26}$

$\boxed{\frac{1000\sqrt{26}}{13} \text{ mi/hR}}$



$s^2 = 5^2 + 1^2$

$= 25 + 1$

$s^2 = 26$

$s = \sqrt{26}$

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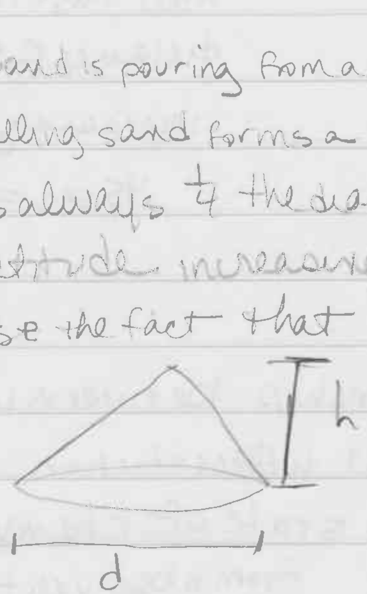
1-2:15 pm

(9) Sand is pouring from a pipe at the rate of 16 ft^3/sec. If the falling sand forms a conical pile on the ground whose altitude is always $\frac{1}{4}$ the diameter of the base, how fast is the altitude increasing when the pile is 4 ft high?

Use the fact that $V=\frac{1}{3}\pi r^2 h$. (Find dh/dt so get V in terms of h.



$h = \frac{1}{4}d \qquad d = 2r$

$= \frac{1}{4}2r$

$h = \frac{r}{2} \qquad r = 2h$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi (2h)^2 h$$

$$V = \frac{4}{3}\pi h^3$$ relationship in V & h

implicit differentiation

$$\frac{dV}{dt} = 3\cdot\frac{4}{3}\pi h^2 \cdot \frac{dh}{dt}$$

$$16 = 4\pi(4)^2 \cdot \frac{dh}{dt}$$

$$16 = 64\pi \frac{dh}{dt}$$

$$\frac{16}{64\pi} = \frac{dh}{dt}$$

$$\frac{1}{4\pi} = \frac{dh}{dt}$$

$$.0796 \frac{ft}{sec} = dh/dt$$