3.8 Related Rates

A student is using a straw to drink from a conical paper cup, whose axis is vertical, at a rate of 3 cm³/sec. If the height of the cup is 10 cm and the diameter of its opening is 6 cm, how fast is the level of the liquid falling when the depth of the liquid is 5 cm?

Volume of cone \( V = \frac{1}{3} \pi r^2 h \) ← make into 2 variables \( V \) and \( h \) for the facts we have and want to find.

Height of the water in cone is always parallel to other heights, the vertex of the cone, axis, and side form a triangle

And any height of water will form similar triangles \( \triangle ABC \sim \triangle DEC \) with corresponding proportional sides \( \frac{r}{h} = \frac{3}{10} \rightarrow \frac{3/10}{h} \)
\[ V = \frac{1}{3} \pi r^2 h \quad \frac{r}{h} = \frac{3}{10} \]
\[ r = \frac{3}{10} h \]
\[ V = \frac{1}{3} \pi \left( \frac{3}{10} h \right)^2 \cdot h \]

\[ V = \frac{3\pi}{100} h^3 \]

\[ \frac{dV}{dt} = 3 \cdot 3\pi h^2 \cdot \frac{dh}{dt} \]

Now evaluate at \( \frac{dV}{dt} = 3 \), \( h = 5 \)

Then solve for \( \frac{dh}{dt} \)

\[ 3 = \frac{9\pi}{100} \cdot 25 \cdot \frac{dh}{dt} \]

\[ 3 = \frac{9\pi}{4} \cdot \frac{dh}{dt} \]

\[ \frac{3 \cdot 4}{9\pi} = \frac{dh}{dt} \]

\[ \frac{4}{3\pi} = \frac{dh}{dt} \]

Answer: \( A + h = 5 \)

height of water is falling \( \frac{4}{3\pi} \) cm/sec

\( \approx 0.42 \) cm/sec
3.8 Related Rates

How fast is the airplane distance from the observer increasing 45 see later?

\[ \frac{dx}{dt} = 400 \text{ mi/hr} \]

\[ x^2 + 1^2 = s^2 \]

relationship between variables.

implicit differentiation to find

\[ \frac{ds}{dt} = \text{rate of change of distance from observer to airplane} \]

\[ 2x \cdot \frac{dx}{dt} + 0 = 2s \cdot \frac{ds}{dt} \]

\[ 2x \frac{dx}{dt} = 2s \frac{ds}{dt} \]

\[ \frac{x}{s} \frac{dx}{dt} = \frac{ds}{dt} \]

at \( t = \frac{1}{80} \text{ hr} \)

\[ \frac{5 \cdot 400}{\sqrt{120}} = \frac{ds}{dt} \]

\( X = \text{distance airplane travelled} \)

\( = 400 \times \frac{1}{80} \)

\[ = \frac{400}{80} \]

\( = \frac{400}{80} \)

\( X = s = 5 \text{ mi} \)

\( \sqrt{120} s \)

\( 1 \text{ mi} \)

\[ s^2 = s^2 + 1^2 \]

\[ = 25 + 1 \]

\[ s^2 = 26 \]

\[ s = \sqrt{26} \]

1-2:15 pm
(9) Sand is pouring from a pipe at the rate of $18 \text{ ft}^3/\text{sec}$. If the falling sand forms a conical pile on the ground whose altitude is always \( \frac{1}{4} \) the diameter of its base, how fast is the altitude increasing when the pile is 4 ft high?

Use the fact that \( V = \frac{1}{3} \pi r^2 h \). (Find \( \frac{dh}{dt} \) so get \( V \) in terms of \( h \).)

\[
V = \frac{1}{3} \pi r^2 h
\]

\[
= \frac{1}{3} \pi (2h)^2 h
\]

\[
V = \frac{4}{3} \pi h^3
\]

**relationship in \( V \) & \( h \).**

**implicit differentiation**

\[
\frac{dV}{dt} = 3 \cdot \frac{4}{3} \pi h^2 \cdot \frac{dh}{dt}
\]

\[
160 = 4 \pi (4)^2 \cdot \frac{dh}{dt}
\]

\[
160 = 16^2 \pi \frac{dh}{dt}
\]

\[
\frac{160}{16 \pi} = \frac{dh}{dt}
\]

\[
\frac{1}{4 \pi} = \frac{dh}{dt}
\]

\[
0.049 \frac{ft}{sec} = \frac{dh}{dt}
\]