Implicit Differentiation

3.7

(2) Find $D_x y$ by implicit differentiation.

$$9x^2 + 4y^2 = 36$$

$$18x + 8y \cdot \frac{dy}{dx} = 0$$

$$8y \cdot \frac{dy}{dx} = -18x$$

$$\frac{dy}{dx} = \frac{-18x}{8y}$$

$$\boxed{\frac{dy}{dx} = -\frac{9x}{4y}}$$

(12) $\cos(xy^2) = y^2 + x$ (Chain Rule + product rule on xy^2)

$$-\sin(xy^2)\left(y^2 + x \cdot 2y \frac{dy}{dx}\right) = 2y \cdot \frac{dy}{dx} + 1$$

$$-y^2 \sin(xy^2) - 2xy \sin(xy^2) \frac{dy}{dx} = 2y \frac{dy}{dx} + 1$$

$$-1\left(y^2 \sin(xy^2) + 1\right) = \left[2y + 2xy \sin(xy^2)\right] \frac{dy}{dx}$$

$$\boxed{\frac{-\left(y^2 \sin(xy^2) + 1\right)}{2y\left(1 + x \sin(xy^2)\right)} = \frac{dy}{dx}}$$

(20) Find $\frac{dy}{dx}$

$$y = \sqrt[3]{x} - 2x^{7/2}$$

$$y = x^{1/3} - 2x^{7/2}$$

$$\frac{dy}{dx} = \frac{1}{3}x^{1/3-1} - \frac{7}{2}\cdot 2x^{7/2-1}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{3}x^{-2/3} - 7x^{5/2}}$$

(28) Find $\frac{dy}{dx}$ $\quad y = \sqrt{x^2\cos x}$

$$y = \sqrt{x^2\cos x} = (x^2\cos x)^{1/2}$$

Chain Rule (Inside Product Rule)

$$\frac{dy}{dx} = \frac{1}{2}(x^2\cos x)^{1/2-1}(2x\cos x + x^2(-\sin x))$$

$$\boxed{\frac{dy}{dx} = \frac{2x\cos x - x^2\sin x}{2\sqrt{x^2\cos x}}}$$