

Differentials and Approximations

3.11

Find dy

(2)

$$y = 7x^3 + 3x^2 + 1$$

$$dy = (21x^2 + 6x) dx$$

(6) Find dy $y = (\tan x + 1)^3$

$$dy = 3(\tan x + 1)^2 \cdot \sec^2 x dx$$

$$dy = 3\sec^2 x (\tan x + 1)^2 dx$$

(10) Let $y = f(x) = x^3$. Find the value of dy in each case

$$y = x^3$$
$$dy = 3x^2 dx$$

a) $x = 0.5, dx = 1$

$$dy = 3x^2 dx$$
$$= 3(0.5)^2 (1)$$
$$= 3(0.25) = \boxed{0.75}$$

b) $x = -1, dx = 0.75$

$$dy = 3x^2 dx$$
$$= 3(-1)^2 (0.75)$$
$$= \boxed{2.25}$$

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(18) Use differentials to approximate the given number. Compare with calculator values

$$\sqrt{402} = \sqrt{x + \Delta x} = \sqrt{400 + 2}$$

$$x = 400, \quad \Delta x = 2, \quad f(x) = \sqrt{x}$$

$$dx = 2$$

$$f(x + \Delta x) \approx f(x) + dy$$

$$\approx f(x) + f'(x)dx$$

$$f(x) = \sqrt{x} = (x)^{1/2}$$

$$f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$f(402) = f(400 + 2) = f(x + \Delta x) \approx f(x) + f'(x)dx$$

$$\approx f(400) + f'(400) \cdot 2$$

$$\approx \sqrt{400} + \frac{1}{2\sqrt{400}} \cdot 2$$

$$= 20 + \frac{1}{20}$$

$$\boxed{\sqrt{402} \approx 20.05}$$

calculator : 20.04993765...