Derivatives of Trigonometric Functions

3.4 Find $D_x y$

(1) $y = 2\sin x + 3\cos x$

$D_x y = 2\cos x + 3\cdot(-\sin x)$

$\boxed{D_x y = 2\cos x - 3\sin x}$

(2) $y = \sin^2 x$

$= \underbrace{\sin x}_{f} \cdot \underbrace{\sin x}_{g}$

$f = \sin x \quad g = \sin x$

$f' = \cos x \quad g' = \cos x$

$y' = f'\cdot g + f\cdot g'$

$= (\cos x)(\sin x) + (\sin x)(\cos x)$

$\boxed{y' = 2\sin x\cos x}$

(4) $y = 1 - \cos^2 x$

$y' = 0 - [\cos x \cdot \cos x]'$

$= -[-2\sin x\cos x]$

$\boxed{y' = 2\sin x\cos x}$

$\underbrace{\cos x}_{f} \cdot \underbrace{\cos x}_{g}$

$f = \cos x \quad g = \cos x$

$f' = -\sin x \quad g' = -\sin x$

$[\cos x\cdot\cos x]' = f'\cdot g + f\cdot g'$

$= (-\sin x)(\cos x) + (\cos x)(-\sin x)$

$= -2\sin x\cos x$

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(6) $y = \csc x = \frac{1}{\sin x} = \frac{f}{g}$ $\quad f=1 \quad g=\sin x$ $\quad f'=0 \quad g'=\cos x$

$$\left[\frac{f}{g}\right]' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$= \frac{0 \cdot \sin x - 1 \cdot \cos x}{(\sin x)^2}$$

$$= \frac{1}{\sin x} \cdot \frac{-\cos x}{\sin x}$$

$$= \boxed{-\csc x \cdot \cot x}$$

(10) $y = \frac{\sin x + \cos x}{\tan x} = \frac{f}{g}$ $\quad f = \sin x + \cos x$ $\quad f' = \cos x - \sin x$ $\quad g = \tan x$ $\quad g' = \sec^2 x$

$$\left[\frac{f}{g}\right]' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$= \frac{(\cos x - \sin x)\tan x - (\sin x + \cos x)\cdot \sec^2 x}{\tan^2 x}$$

$$= \frac{\cos x \tan x - \sin x \cdot \tan x - \sin x \cdot \sec^2 x + \cos x \cdot \sec^2 x}{\tan^2 x}$$

$$= \frac{\cos x \tan x}{\tan^2 x} - \frac{\sin x \tan x}{\tan^2 x} - \frac{\sin x \cdot \sec^2 x}{\tan^2 x} - \frac{\cos x \sec^2 x}{\tan^2 x}$$

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(10) continued $y = \dfrac{\sin x + \cos x}{\tan x} = \dfrac{f}{g}$

...

$$\left[\frac{f}{g}\right]' = \frac{\cos x \tan x}{\tan^2 x} - \frac{\sin x \tan x}{\tan^2 x} - \frac{\sin x \sec^2 x}{\tan^2 x} - \frac{\cos x \sec^2 x}{\tan^2 x}$$

$$= \frac{\cos x}{\tan x} - \frac{\sin x}{\tan x} - \frac{\sin x}{\tan^2 x \cdot \cos^2 x} - \frac{\cos x}{\tan^2 x \cdot \cos^2 x}$$

$$= \frac{\cos x}{\sin x} \cdot \cos x - \frac{\cos x}{\sin x} \cdot \sin x - \frac{\cos^2 x}{\sin^2 x} \cdot \frac{\sin x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} \cdot \frac{\cos x}{\cos^2 x}$$

$$= \frac{\cos x \cdot \cos x}{\sin x} - \cos x - \frac{1}{\sin x} - \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$$

$$= \cot x \cdot \cos x - \cos x - \csc x - \csc x \cdot \cot x$$