Derivatives of Exponential and Logarithmic Functions.

3.9/

(2) show that f has an inverse by showing that it is strictly monotonic.

$$f(x) = x^7 + x^5 + x^3 + x$$

$$f'(x) = 7x^6 + 5x^4 + 3x^2 + 1$$

all even terms
each term > 0 for all x

$\therefore f'(x) > 0$ for all x.

$\therefore f(x)$ is strictly monotonic INCREASING

and therefore one-to-one.

Therefore it has an inverse

(20) Find $\frac{dy}{dx}$ if $y = x^2 \ln x = \underbrace{x^2}_{f} \cdot \underbrace{\ln x}_{g}$

$f = x^2 \quad g = \ln x$

$f' = 2x \quad g' = 1/x$

$$\frac{dy}{dx} = f' \cdot g + f \cdot g'$$

$$= 2x \cdot \ln x + x^2 \cdot \frac{1}{x}$$

$$\boxed{\frac{dy}{dx} = 2x \ln x + x}$$

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(14) Find $(f^{-1})'(2)$ using $(f^{-1})'(y) = \frac{1}{f'(x)}$

for $f(x) = \sqrt{x+1}$

when $\boxed{y = 2}$

$$f(x) = \sqrt{x+1}$$
$$2 = \sqrt{x+1}$$
$$4 = x+1$$
$$3 = x$$

Therefore, $(f^{-1})'(2) = \frac{1}{f'(3)}$

$$f(x) = \sqrt{x+1} = (x+1)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(x+1)^{-1/2}$$

$$f'(x) = \frac{1}{2\sqrt{x+1}}$$

$$f'(3) = \frac{1}{2\sqrt{3+1}} = \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2} = \frac{1}{4}$$

$$(f^{-1})'(2) = \frac{1}{f'(3)} = \frac{1}{\frac{1}{4}} = 4$$

$$\therefore \boxed{(f^{-1})'(2) = 4}$$

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16) $f(x) = \ln(3x^3 + 2x)$

$$f'(x) = \frac{1}{3x^3+2x} \cdot (9x^2+2)$$

$$\boxed{f'(x) = \frac{9x^2+2}{3x^3+2x}}$$

18) $y = \ln\sqrt{3x-2}$

$$y = \ln(3x-2)^{1/2}$$

$$y = \frac{1}{2}\ln(3x-2)$$

$$y' = \frac{1}{2} \cdot \frac{1}{3x-2} \cdot 3$$

$$\boxed{y' = \frac{3}{2(3x-2)}}$$

OR

$$y = \ln(3x-2)^{\frac{1}{2}}$$

$$y' = \frac{1}{(3x-2)^{1/2}} \cdot \frac{1}{2}(3x-2)^{-1/2} \cdot 3$$

$$y' = \frac{3}{2(3x-2)^{1/2}(3x-2)^{1/2}}$$

$$y' = \frac{3}{2(3x-2)^1}$$

$$\boxed{y' = \frac{3}{2(3x-2)}}$$

20) $y = \underbrace{x^2}_{f} \cdot \underbrace{\ln x}_{g}$

$f = x^2$ $\quad g = \ln x$

$f' = 2x$ $\quad g' = \frac{1}{x}$

$$y' = f' \cdot g + f \cdot g'$$

$$= (2x)\ln x + x^2 \cdot \frac{1}{x}$$

$$\boxed{y' = 2x\ln x + x}$$

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(26) Find $f'(\pi/4)$ if $f(x) = \ln(\cos x)$

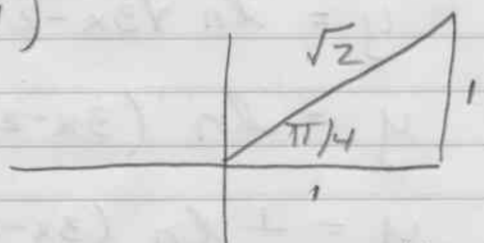
$$f'(x) = \frac{1}{\cos x} \cdot -\sin x$$

$$= -\frac{\sin x}{\cos x}$$

$$f'(x) = -\tan x$$

$$f'(\pi/4) = -\tan(\pi/4)$$

$$\boxed{f'\left(\tfrac{\pi}{4}\right) = -1}$$



$\tan(\pi/4) = \frac{1}{1} = 1$

(28) $y = e^{2x^2 - x}$

$$\frac{dy}{dx} = e^{2x^2-x}(4x-1)$$

$$\boxed{\frac{dy}{dx} = (4x-1) \cdot e^{2x^2-x}}$$

(30) $y = e^{-1/x^2} = e^{-x^{-2}}$

$$y' = e^{-x^{-2}} \cdot (-2 \cdot -x^{-3})$$

$$\boxed{y' = \frac{2e^{-1/x^2}}{x^3}}$$

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(34) $y = e^{x^3 \ln x}$

$y' = e^{x^3 \ln x} \cdot \left(3x^2 \cdot \ln x + x^3 \cdot \frac{1}{x}\right)$

$$\boxed{y' = e^{x^3 \ln x}\left(3x^2 \ln x + x^2\right)}$$

(and you can even factor out the x^2 if you want!)

(40) $y = 3^{2x^2 - 3x}$

(Example of log differentiation method.)

$\ln y = \ln 3^{2x^2 - 3x}$

$\ln y = (2x^2 - 3x) \cdot \ln 3$

$\frac{1}{y} \cdot \frac{dy}{dx} = (4x - 3) \ln 3$

$\frac{dy}{dx} = y \cdot (4x - 3) \cdot \ln 3$

$= 3^{2x^2 - 3x} \cdot (4x - 3) \cdot \ln 3$

$$\boxed{\frac{dy}{dx} = (4x - 3) \cdot 3^{2x^2 - 3x} \cdot \ln 3}$$