Derivatives of Hyperbolic and Inverse Trigonometric Functions.

3.10

(2) $y = \cosh^2 x = (\cosh x)^2$

$y' = 2(\cosh x)^{2-1} \cdot \sinh x$

$\boxed{y' = 2\cosh x \sinh x}$ OR $= \sinh 2x$.

(6) $y = \sinh(x^2+x)$

$\frac{dy}{dx} = \cosh(x^2+x) \cdot (2x+1)$

$\boxed{\frac{dy}{dx} = (2x+1)\cosh(x^2+x)}$

(8) $y = \ln(\coth x)$

$D_x y = \frac{1}{\coth x} \cdot -\operatorname{csch}^2 x$

$= -\frac{\sinh x}{\cosh x} \cdot \frac{1}{\sinh^2 x}$

$\boxed{D_x y = \frac{-1}{\sinh x \cosh x}}$

(26) $y = \arccos(e^x) = \cos^{-1}(e^x)$

$$D_x y = -\frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x \quad \text{(Chain Rule!)}$$

$$\boxed{D_x y = \frac{-e^x}{\sqrt{1-e^{2x}}}}$$

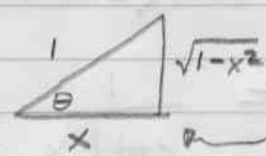
(30) $y = \tan(\cos^{-1} x)$

$$y' = \underbrace{\sec^2(\cos^{-1} x)} \cdot -\frac{1}{\sqrt{1-x^2}}$$

BTW: $\sec^2(\cos^{-1} x) = \sec^2\theta = (\sec\theta)^2 = \left(\frac{HYP}{ADJ}\right)^2 = \left(\frac{1}{x}\right)^2 = \frac{1}{x^2}$

let $\theta = \cos^{-1} x$

$\Rightarrow \cos\theta = x = \frac{x}{1} = \frac{AD}{HYP}$



$\rightarrow \sec\theta = \frac{HYP}{AD} = \frac{1}{x}$

$$\therefore y' = \sec^2(\cos^{-1} x) \cdot \frac{-1}{\sqrt{1-x^2}}$$

$$= \frac{1}{x^2} \cdot \frac{-1}{\sqrt{1-x^2}}$$

$$\boxed{y' = \frac{-1}{x^2\sqrt{1-x^2}}}$$

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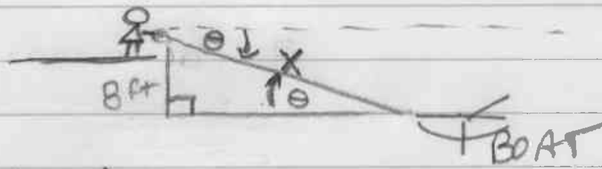
3.10 (Related Rate PROBLEM)

(44) A man on a dock is pulling in a rope attached to a rowboat at a rate of 5 ft per second. If the man's hands are 8 ft higher than the point where the rope is attached to the boat, how fast is the angle of depression of the rope changing when there are still 17 feet of rope out?



angle of depression θ is ~~alternate~~ interior angle to Δ. and equal.

To Find $d\theta/dt$

Describe relationship with θ

length of rope = Hypotenuse = x

$$\sin\theta = \frac{8}{x}$$

$$\theta = \sin^{-1}\left(\frac{8}{x}\right)$$

* Find $\frac{d\theta}{dt}$ when $x = 17$ ft

$\frac{dx}{dt} = -5$

$$\frac{d\theta}{dt} = \frac{1}{\sqrt{1-\left(\frac{8}{x}\right)^2}} \cdot \frac{-8}{x^2} \cdot \frac{dx}{dt}$$

$$= \frac{-8}{x^2\sqrt{\frac{x^2}{x^2} - \frac{64}{x^2}}}\frac{dx}{dt} = \frac{-8}{x^2\frac{\sqrt{x^2-64}}{\sqrt{x^2}}}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{-8}{x\sqrt{x^2-64}} \cdot \frac{dx}{dt}$$

when $x = 17$

$\frac{dx}{dt} = -5$

$$\frac{d\theta}{dt} = \frac{-8}{17\sqrt{17^2-64}} \cdot -5 = \frac{40}{17\sqrt{225}}$$

$$= \frac{40}{17 \cdot 15} = \frac{8}{51}$$

$$\boxed{d\theta/dt = \frac{8}{51} \text{ radians per sec}}$$