The Chain Rule

3.5

$$\boxed{\begin{aligned} y &= f(g(x)) \\ y' &= f'(g(x)) \cdot g'(x) \end{aligned}}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$ Leibnitz notation

Find $D_x y$

4 $y = (4+2x^2)^7$ $f(x) = x^7$ $g(x) = 4+2x^2$

$$y' = 7(4+2x^2)^{7-1} \cdot (0+4x^{2-1})$$
$$= 7(4+2x^2)^6 (4x)$$
$$\boxed{y' = 28x(4+2x^2)^6}$$

6 $y = (x^2 - x + 1)^{-7}$

$$y' = \underbrace{-7(x^2-x+1)^{-7-1}}_{f'(g(x))} \cdot \underbrace{(2x-1+0)}_{g'(x)}$$

D_x (outside) $\cdot$ D_x (inside)

$$\boxed{y' = -7(2x-1)(x^2-x+1)^{-8}}$$

8 $y = \dfrac{1}{(3x^2+x-3)^9} = (3x^2+x-3)^{-9}$

$$y' = -9(3x^2+x-3)^{-9-1} \cdot (6x+1-0)$$
$$= -9(6x+1)(3x^2+x-3)^{-10}$$
$$\boxed{y' = \frac{-9(6x+1)}{(3x^2+x-3)^{10}}}$$

The Chain Rule

Find the derivative using the Chain Rule

10

$y = \cos(3x^2 - 2x) = f(g(x))$ where

$f(x) = \cos x \quad f'(x) = -\sin x$

$g(x) = 3x^2 - 2x$

$y' = \underbrace{-\sin(3x^2 - 2x)}_{f'(g(x))} \cdot \underbrace{(6x - 2)}_{g'(x)}$

$y' = -(6x - 2)\sin(3x^2 - 2x)$

$\boxed{y' = (2 - 6x)\sin(3x^2 - 2x)}$

12 $y = \sin^4(3x^2)$

$= [\sin(3x^2)]^4$ outside $= x^4$ inside $= \sin(3x^2)$

$D_x(\text{inside}) =$

$\cos(3x^2) \cdot 6x$

$= 6x \cdot \cos(3x^2)$

$= 4[\sin(3x^2)]^{4-1} \cdot \cos(3x^2) \cdot 6x$

$\boxed{y' = 24x \cos(3x^2) \sin^3(3x^2)}$

3.5 The Chain Rule

Find the derivative using the Chain Rule

(22) $y = (x+\sin x)^2$

$y' = 2(x+\sin x)^{2-1} \cdot (1+\cos x)$

$\boxed{y' = 2(x+\sin x)(1+\cos x)}$

(30) Find $G'(1)$ if $G(t) = (t^2+9)^3 \cdot (t^2-2)^4$

$G(t) = \underbrace{(t^2+9)^3}_{f} \cdot \underbrace{(t^2-2)^4}_{g}$

$G'(t) = f' \cdot g + f \cdot g'$

$f = (t^2+9)^3$

$f' = 3(t^2+9)^2 \cdot (2t)$

$g = (t^2-2)^4$

$g' = 4(t^2-2)^3 \cdot 2t$

$= 8t(t^2-2)^3$

$G'(t) = f' \cdot g + f \cdot g'$

$= [6t(t^2+9)^2] \cdot [t^2-2]^4 + (t^2+9)^3 \cdot 8t(t^2-2)^3$

So

$G'(1) = [6(1+9)^2] \cdot (1-2)^4 + (1+9)^3 \cdot 8(1-2)^3$

$= [600] \cdot 1 + 8000 \cdot (-1)$

$= 600 - 8000$

$= -7400$

The Chain Rule 3.5

(62) Find the equation of the tangent line to the graph of $y = 1 + x\sin 3x$ at $(\pi/3, 1)$. Where does the line cross the x-axis?

equation of tangent line $y - y_1 = m(x - x_1)$
at $\left(\frac{\pi}{3}, 1\right) = (x_1, y_1)$

Find slope $m = y'$ evaluated at $x = \pi/3$

$y = 1 + \underbrace{x}_{f} \cdot \underbrace{\sin 3x}_{g}$

Product Rule
$f = x \quad g = \sin 3x$
$f' = 1 \quad g' = 3\cos 3x$

$y' = 0 + f' \cdot g + f \cdot g'$

$= 1 \cdot \sin 3x + x \cdot 3\cos 3x$

$y' = \sin(3x) + 3x \cdot \cos(3x)$

at $x = \pi/3$ $y' = \sin\left(3 \cdot \frac{\pi}{3}\right) + 3 \cdot \frac{\pi}{3} \cdot \cos\left(3 \cdot \frac{\pi}{3}\right)$

$= \sin(\pi) + \pi \cdot \cos(\pi)$

$= 0 + \pi \cdot (-1)$

$y' = -\pi$ slope $= -\pi$

$(x_1, y_1) = (\pi/3, 1)$
$m = -\pi$

$y - y_1 = m(x - x_1)$

equation of tangent line at $x = \pi/3$

$y - 1 = -\pi(x - \pi/3)$

$y = -\pi x + \frac{\pi^2}{3} + 1$ OR $\boxed{y = -\pi x + \frac{\pi^2 + 3}{3}}$