(2) Find the limit

$$\lim_{x \to \infty} \frac{x^2}{5 - x^3} = \lim_{x \to \infty} \frac{\cancel{x^3} \cdot \frac{1}{x}}{\cancel{x^3}\left(\frac{5}{x^3} - 1\right)}$$

factor highest power of x in denominator!

$$= \lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{5}{x^3} - 1}$$

Since $\lim_{x \to \infty} \frac{1}{x} = \frac{1}{\infty} \to 0$

and Thm A

$$\lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{5}{x^3} - 1} = \frac{0}{0-1} = \frac{0}{-1}$$

$$= \boxed{0}$$

(8) $$\lim_{\theta \to -\infty} \frac{\pi \theta^5}{\theta^5 - 5\theta^4} = \lim_{\theta \to \infty} \frac{\cancel{\theta^5} \cdot \pi}{\cancel{\theta^5}\left(1 - \frac{5}{\theta}\right)}$$

$$= \lim_{\theta \to -\infty} \frac{\pi}{1 - \frac{5}{\theta}} = \frac{\pi}{1-0}$$

$$= \frac{\pi}{1} = \boxed{\pi}$$

(10) $\lim\limits_{\theta \to \infty} \dfrac{\sin^2\theta}{\theta^2 - 5}$

$\forall \theta \quad 0 \le \sin^2\theta \le 1$

$\therefore \quad \dfrac{\sin^2\theta}{\theta^2-5} \le \dfrac{1}{\theta^2-5}$ for all θ.

Since $\lim\limits_{\theta \to \infty} \dfrac{1}{\theta^2-5} = \lim\limits_{\theta \to \infty} \dfrac{\cancel{\theta^2}\left(\frac{1}{\theta^2}\right)}{\cancel{\theta^2}\left(1-\frac{5}{\theta^2}\right)}$

$= \lim\limits_{\theta \to \infty} \dfrac{\frac{1}{\theta^2}}{1-\frac{5}{\theta^2}} = \dfrac{0}{1-0}$

$= \dfrac{0}{1}$

$= 0$

Since $\dfrac{\sin^2\theta}{\theta^2-5} \le \dfrac{1}{\theta^2-5}$ for all θ

in particular all $\theta \to \infty$

and $\lim\limits_{\theta \to \infty} \dfrac{1}{\theta^2-5} = 0$

then $\lim\limits_{\theta \to \infty} \dfrac{\sin^2\theta}{\theta^2-5} = \boxed{0}$

2.4 Limits at Infinity

(14) $\lim_{x\to\infty} \sqrt{\dfrac{x^2+x+3}{(x-1)(x+1)}} = \lim_{x\to\infty} \sqrt{\dfrac{x^2+x+3}{x^2-1}}$

$= \lim_{x\to\infty} \sqrt{\dfrac{x^2\left(1+\frac{1}{x}+3/x^2\right)}{x^2\left(1-1/x^2\right)}}$

$= \lim_{x\to\infty} \sqrt{\dfrac{1+1/x+3/x^2}{1-1/x^2}}$

$= \sqrt{\dfrac{1+0+0}{1-0}}$

$= \sqrt{\dfrac{1}{1}} = \boxed{1}$

(22) $\lim_{x \to \infty} \left(\sqrt{x^2+2x} - x \right)$

$$= \lim_{x \to \infty} \frac{\left(\sqrt{x^2+2x} - x\right) \cdot \left(\sqrt{x^2+2x} + x\right)}{\left(\sqrt{x^2+2x} + x\right)}$$

$$= \lim_{x \to \infty} \frac{(x^2+2x) - x^2}{\sqrt{x^2+2x} + x}$$

$$= \lim_{x \to \infty} \frac{2x}{\sqrt{x^2+2x} + x}$$

skip
more advanced students

$$= \lim_{x \to \infty} \frac{x \cdot 2}{\sqrt{x^2(1+2/x)} + x}$$

$$= \lim_{x \to \infty} \frac{x \cdot 2}{\sqrt{x^2} \cdot \sqrt{(1+2/x)} + x}$$

$\sqrt{x^2} = x$

$$= \lim_{x \to \infty} \frac{\cancel{x} \cdot 2}{\cancel{x}\left(\sqrt{1+2/x} + 1\right)}$$

$$= \lim_{x \to \infty} \frac{2}{\sqrt{1+2/x} + 1}$$

$$= \frac{2}{\sqrt{1+0} + 1}$$

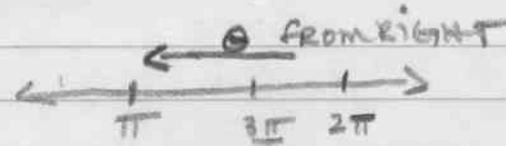
$$= \frac{2}{\sqrt{1} + 1} = \frac{2}{2} = \boxed{1}$$

2.4

(32) $\lim_{\theta \to \pi^+} \dfrac{\theta^2}{\sin\theta} = \boxed{-\infty}$ proof follows.

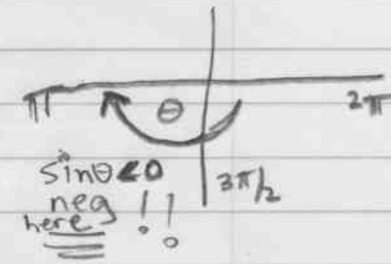
as $\theta \to \pi^+$ $\theta^2 \to \pi^2$ positive

as $\theta \to \pi^+$



$\sin\theta \to 0^-$ from neg values of $\sin\theta$

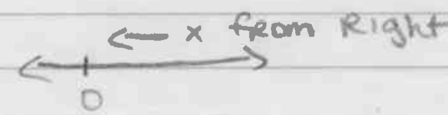
hence from left of 0



Therefore $\lim_{\theta \to \pi^+} \dfrac{\theta^2}{\sin\theta} \to \dfrac{\pi^2}{\text{neg values approaching } 0} \to -\infty$

(40) $\lim_{x \to 0^+} \dfrac{|x|}{x}$

as $x \to 0^+$ $x > 0$



and $|x| = x$ for all $x > 0$

So $\lim_{x \to 0^+} \dfrac{|x|}{x} = \lim_{x \to 0^+} \dfrac{x}{x} = \lim_{x \to 0^+} 1 = \boxed{1}$