2.3

Justify each step to finding the limit of each function. using theorem A.

(2) $\lim_{x \to -1} (3x^2 - 1) \overset{(5)}{=} \lim_{x \to -1} 3x^2 - \lim_{x \to -1} 1$

$\overset{(3)}{=} 3 \lim_{x \to -1} x^2 - \lim_{x \to -1} 1$

$\overset{(8)}{=} 3 \left(\lim_{x \to -1} x\right)^2 - \lim_{x \to -1} 1$

$\overset{(2)}{=} 3 \cdot (-1)^2 - \lim_{x \to -1} 1$

$\overset{(1)}{=} 3 \cdot (-1)^2 - 1$

$= 3 - 1$

$= \boxed{2}$

2.3

(10) Use Thm A and the numbered statements to justify each step in finding the limit of the function.

$$\lim_{w \to -2} \sqrt{-3w^3 + 7w^2} \overset{9}{=} \sqrt{\lim_{w \to -2} (-3w^3 + 7w^2)}$$

$$\overset{4}{=} \sqrt{\lim_{w \to -2} -3w^3 + \lim_{w \to -2} 7w^2}$$

$$\overset{3}{=} \sqrt{-3 \lim_{w \to -2} w^3 + 7 \lim_{w \to -2} w^2}$$

$$\overset{8}{=} \sqrt{-3\left(\lim_{w \to -2} w\right)^3 + 7\left(\lim_{w \to -2} w\right)^2}$$

$$\overset{2}{=} \sqrt{-3 \cdot (-2)^3 + 7(-2)^2}$$

$$= \sqrt{-3(-8) + 7(4)}$$

$$= \sqrt{24+28}$$

$$= \sqrt{52}$$

$$= \sqrt{4 \cdot 13}$$

$$= \sqrt{4}\sqrt{13} = \boxed{2\sqrt{13}}$$

2.3 Limit Theorems

(16) Find the limit or state that it does not exist.

$$\lim_{x \to -1} \frac{x^2+x}{x^2+1} = \frac{(-1)^2+(-1)}{(-1)^2+1}$$

$$= \frac{1+(-1)}{1+1} = \frac{0}{2} = \boxed{0}$$

(20) $$\lim_{x \to -3} \frac{x^2-14x-51}{x^2-4x-21}$$

$$= \lim_{x \to -3} \frac{\cancel{(x+3)}(x-17)}{\cancel{(x+3)}(x-7)}$$

$$= \lim_{x \to -3} \frac{x-17}{x-7} = \frac{-3-17}{-3-7} = \frac{-20}{-10} = \boxed{2}$$

(22) $$\lim_{x \to 1} \frac{x^2+ux-x-u}{x^2+2x-3} = \lim_{x \to 1} \frac{(x+u)\cancel{(x-1)}}{(x+3)\cancel{(x-1)}}$$

$$= \lim_{x \to 1} \frac{x+u}{x+3}$$

$$= \frac{1+u}{1+3} = \boxed{\frac{1+u}{4}}$$

Footnote ....

* Factor By Grouping: $x^2+ux-x-u$
(Review)

$$= (x^2+ux)-(x+u)$$
$$= x(x+u)-1\cdot(x+u)$$
$$= (x+u)(x-1)$$

2.3 Limit Theorems

26 Find the limit if $\lim_{x \to a} f(x) = 3$

$\lim_{x \to a} g(x) = -1$

$$\lim_{x \to a} \frac{2f(x) - 3g(x)}{f(x) + g(x)}$$

$$= \frac{2 \cdot 3 - 3 \cdot (-1)}{3 + (-1)}$$

$$= \frac{6 + 3}{2} = \boxed{\frac{9}{2}}$$

2.3 Limit Theorems

(32) Find $\lim_{x \to 2} \dfrac{[f(x) - f(2)]}{(x-2)}$ for the given $f(x)$.

$f(x) = 3x^2 + 2x + 1$

$f(2) = 3(2)^2 + 2(2) + 1 = 3 \cdot 4 + 2 \cdot 2 + 1 = 17$

$[f(x) - f(2)] = (3x^2 + 2x + 1) - 17$

$= 3x^2 + 2x - 16$

$\lim_{x \to 2} \dfrac{[f(x) - f(2)]}{(x-2)} = \lim_{x \to 2} \dfrac{3x^2 + 2x - 16}{(x-2)}$

$= \lim_{x \to 2} \left(\dfrac{\cancel{(x-2)}(3x+8)}{\cancel{(x-2)}} \right)$

$= \lim_{x \to 2} (3x + 8) = 3(2) + 8$

$= 6 + 8$

$= \boxed{14}$

(43) Find the one-sided limit or state it does not exist

$\lim_{x \to 3^+} \dfrac{(x-3)}{\sqrt{x^2-9}} = \lim_{x \to 3^+} \dfrac{(x-3)}{\sqrt{x^2-9}} \cdot \dfrac{\sqrt{x^2-9}}{\sqrt{x^2-9}}$

$x \to 3^+$
$\Rightarrow \sqrt{x^2-9} \geq 0$
so defined!

$= \lim_{x \to 3^+} \dfrac{(x-3) \cdot \sqrt{x^2-9}}{x^2 - 9}$

$= \lim_{x \to 3^+} \dfrac{\cancel{(x-3)}\sqrt{x^2-9}}{\cancel{(x-3)}(x+3)} = \dfrac{\sqrt{3^2-9}}{3+3} = \dfrac{0}{6}$

$= \boxed{0}$