Natural Exponential, Natural log, and Hyperbolic Functions.

2.6

(2) Simplify the expression.

$$2^{2\log_2 x} = 2^{\log_2 x^2} = \boxed{x^2}$$

(4) $$e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \boxed{\frac{1}{x^2}}$$

(6) $$\ln e^{-2x-3} = \log_e e^{-2x-3} = \boxed{-2x-3}$$

(8) $$e^{x-\ln x} = e^x e^{-\ln x} = \frac{e^x}{e^{\ln x}} = \boxed{\frac{e^x}{x}}$$

(10) $$e^{\ln x^2 - y\ln x} = e^{\ln x^2} e^{-y\ln x}$$

$$= e^{\ln x^2} e^{\ln x^{-y}}$$

$$= x^2 x^{-y}$$

$$= \boxed{x^{2-y}}$$

(24) Write the logarithm as a single quantity or term.

$$\frac{1}{2}\ln(x-9) + \frac{1}{2}\ln x = \ln\sqrt{x-9} + \ln\sqrt{x}$$

$$= \ln\sqrt{x-9}\cdot\sqrt{x}$$

$$= \ln\sqrt{(x-9)x}$$

$$= \boxed{\ln\sqrt{x^2-9x}}$$

(26) $\ln(x^2-9)-2\ln(x-3)-\ln(x+3)$

$=\ln(x^2-9)-\ln(x-3)^2-\ln(x+3)$

$=\ln\dfrac{x^2-9}{(x-3)^2(x+3)}$

$=\ln\dfrac{(x-3)(x+3)}{(x-3)^2(x+3)}$

$=\ln\dfrac{1}{x-3}$

$=\ln(x-3)^{-1}$

$=\boxed{-\ln(x-3)}$

Natural Exponential, Natural Log, and Hyperbolic Functions.

2.6

Verify the identity

(42) $e^{2x} = \cosh 2x + \sinh 2x$

$$\cosh 2x + \sinh 2x = \frac{e^{2x}+e^{-2x}}{2} + \frac{e^{2x}-e^{-2x}}{2}$$

$$= \frac{2e^{2x}}{2} = e^{2x} \quad ✓$$

(46) $\sinh(x-y) = \sinh x \cosh y - \cosh x \sinh y$

$$\sinh x \cosh y - \cosh x \sinh y = \left(\frac{e^{x}-e^{-x}}{2}\right)\cdot\left(\frac{e^{y}+e^{-y}}{2}\right) - \left(\frac{e^{x}+e^{-x}}{2}\right)\cdot\left(\frac{e^{y}-e^{-y}}{2}\right)$$

$$= \frac{e^{x+y}+e^{x-y}-e^{y-x}-e^{-x-y}}{4} - \left(\frac{e^{x+y}-e^{x-y}+e^{y-x}-e^{-x-y}}{4}\right)$$

$$= \frac{\cancel{e^{x+y}}+e^{x-y}-e^{y-x}-e^{-x-y}-\cancel{e^{x+y}}+e^{x-y}-e^{y-x}+e^{-x-y}}{4}$$

$$= \frac{2e^{x-y}-2e^{y-x}}{4} = \frac{2\left(e^{x-y}-e^{y-x}\right)}{4}$$

$$= \frac{e^{x-y}-e^{-(x-y)}}{2}$$

$$= \sinh(x-y) \quad ✓$$

Continuity of Functions

2.7

State whether the indicated function is continuous at 3. If it is not continuous, tell why.

(2) $g(x) = x^2 - 9 \qquad g(3) = 3^2 - 9 = 0$

$\lim_{x \to 3} x^2 - 9 = 0 = g(3)$ $\boxed{\text{continuous at 3}}$

(4) $g(t) = \sqrt{t-4}$

$\lim_{t \to 3} \sqrt{t-4}$ does not exist $\qquad g(3) = \sqrt{-1}$ does not exist

Therefore $\boxed{g(t) \text{ is not continuous at } t = 3.}$

(10) $f(x) = \dfrac{21-7x}{x-3} \qquad f(3)$ does not exist

$\boxed{\therefore f(x) \text{ is not continuous at } x = 3.}$

(12) $r(t) = \begin{cases} \dfrac{t^3-27}{t-3} & t \neq 3 \\ 23 & t = 3 \end{cases}$

$$\lim_{t \to 3} \frac{t^3-27}{t-3} = \lim_{t \to 3} \frac{\cancel{(t-3)}(t^2+3t+9)}{\cancel{(t-3)}} = 9+9+9 = 27$$

Since $\lim_{t \to 3} \dfrac{t^3-27}{t-3} = 27 \neq r(3) = 23$ $\boxed{r(t) \text{ not continuous at } t = 3}$