# NEGATIVE *z* Scores

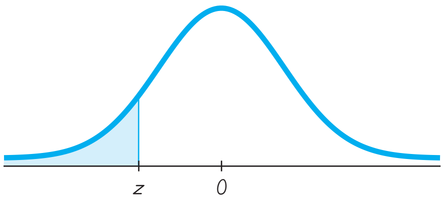
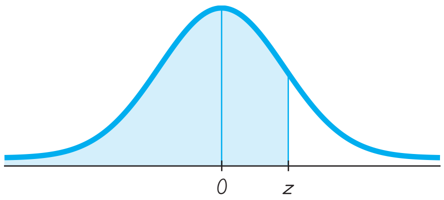



**Table A-2** Standard Normal (*z*) Distribution: Cumulative Area from the LEFT

| *z* | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.50 and lower | .0001 | | | | | | | | | |
| −3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| −3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| −3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| −3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| −3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| −2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| −2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| −2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| −2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| −2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 * | .0049 | .0048 |
| −2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| −2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| −2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| −2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| −2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| −1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| −1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| −1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| −1.6 | .0548 | .0537 | .0526 | .0516 | .0505 * | .0495 | .0485 | .0475 | .0465 | .0455 |
| −1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| −1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| −1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| −1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| −1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| −1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| −0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| −0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| −0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| −0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| −0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| −0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| −0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| −0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| −0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| −0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

*NOTE:* For values of *z* below −3.49, use 0.0001 for the area.
*Use these common values that result from interpolation:

| *z* score | Area |
|---|---|
| −1.645 | 0.0500 |
| −2.575 | 0.0050 |




# POSITIVE *z* Scores

**Table A-2** (*continued*) Cumulative Area from the LEFT

| *z* | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 * | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 * | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |
| 3.50 and up | .9999 | | | | | | | | | |

*NOTE:* For values of *z* above 3.49, use 0.9999 for the area.
*Use these common values that result from interpolation:

| *z* score | Area |
|---|---|
| 1.645 | 0.9500 |
| 2.575 | 0.9950 |

**Common Critical Values**

| Confidence Level | Critical Value |
|---|---|
| 0.90 | 1.645 |
| 0.95 | 1.96 |
| 0.99 | 2.575 |

## Table A-3 *t* Distribution: Critical *t* Values

| | Area in One Tail | | | | |
|---|---|---|---|---|---|
| | 0.005 | 0.01 | 0.025 | 0.05 | 0.10 |
| | Area in Two Tails | | | | |
| Degrees of Freedom | 0.01 | 0.02 | 0.05 | 0.10 | 0.20 |
| 1 | **63.657** | 31.821 | **12.706** | 6.314 | 3.078 |
| 2 | **9.925** | 6.965 | **4.303** | 2.920 | 1.886 |
| 3 | **5.841** | 4.541 | **3.182** | 2.353 | 1.638 |
| 4 | **4.604** | 3.747 | **2.776** | 2.132 | 1.533 |
| 5 | **4.032** | 3.365 | **2.571** | 2.015 | 1.476 |
| 6 | **3.707** | 3.143 | **2.447** | 1.943 | 1.440 |
| 7 | **3.499** | 2.998 | **2.365** | 1.895 | 1.415 |
| 8 | **3.355** | 2.896 | **2.306** | 1.860 | 1.397 |
| 9 | **3.250** | 2.821 | **2.262** | 1.833 | 1.383 |
| 10 | **3.169** | 2.764 | **2.228** | 1.812 | 1.372 |
| 11 | **3.106** | 2.718 | **2.201** | 1.796 | 1.363 |
| 12 | **3.055** | 2.681 | **2.179** | 1.782 | 1.356 |
| 13 | **3.012** | 2.650 | **2.160** | 1.771 | 1.350 |
| 14 | **2.977** | 2.624 | **2.145** | 1.761 | 1.345 |
| 15 | **2.947** | 2.602 | **2.131** | 1.753 | 1.341 |
| 16 | **2.921** | 2.583 | **2.120** | 1.746 | 1.337 |
| 17 | **2.898** | 2.567 | **2.110** | 1.740 | 1.333 |
| 18 | **2.878** | 2.552 | **2.101** | 1.734 | 1.330 |
| 19 | **2.861** | 2.539 | **2.093** | 1.729 | 1.328 |
| 20 | **2.845** | 2.528 | **2.086** | 1.725 | 1.325 |
| 21 | **2.831** | 2.518 | **2.080** | 1.721 | 1.323 |
| 22 | **2.819** | 2.508 | **2.074** | 1.717 | 1.321 |
| 23 | **2.807** | 2.500 | **2.069** | 1.714 | 1.319 |
| 24 | **2.797** | 2.492 | **2.064** | 1.711 | 1.318 |
| 25 | **2.787** | 2.485 | **2.060** | 1.708 | 1.316 |
| 26 | **2.779** | 2.479 | **2.056** | 1.706 | 1.315 |
| 27 | **2.771** | 2.473 | **2.052** | 1.703 | 1.314 |
| 28 | **2.763** | 2.467 | **2.048** | 1.701 | 1.313 |
| 29 | **2.756** | 2.462 | **2.045** | 1.699 | 1.311 |
| 30 | **2.750** | 2.457 | **2.042** | 1.697 | 1.310 |
| 31 | **2.744** | 2.453 | **2.040** | 1.696 | 1.309 |
| 32 | **2.738** | 2.449 | **2.037** | 1.694 | 1.309 |
| 33 | **2.733** | 2.445 | **2.035** | 1.692 | 1.308 |
| 34 | **2.728** | 2.441 | **2.032** | 1.691 | 1.307 |
| 35 | **2.724** | 2.438 | **2.030** | 1.690 | 1.306 |
| 36 | **2.719** | 2.434 | **2.028** | 1.688 | 1.306 |
| 37 | **2.715** | 2.431 | **2.026** | 1.687 | 1.305 |
| 38 | **2.712** | 2.429 | **2.024** | 1.686 | 1.304 |
| 39 | **2.708** | 2.426 | **2.023** | 1.685 | 1.304 |
| 40 | **2.704** | 2.423 | **2.021** | 1.684 | 1.303 |
| 45 | **2.690** | 2.412 | **2.014** | 1.679 | 1.301 |
| 50 | **2.678** | 2.403 | **2.009** | 1.676 | 1.299 |
| 60 | **2.660** | 2.390 | **2.000** | 1.671 | 1.296 |
| 70 | **2.648** | 2.381 | **1.994** | 1.667 | 1.294 |
| 80 | **2.639** | 2.374 | **1.990** | 1.664 | 1.292 |
| 90 | **2.632** | 2.368 | **1.987** | 1.662 | 1.291 |
| 100 | **2.626** | 2.364 | **1.984** | 1.660 | 1.290 |
| 200 | **2.601** | 2.345 | **1.972** | 1.653 | 1.286 |
| 300 | **2.592** | 2.339 | **1.968** | 1.650 | 1.284 |
| 400 | **2.588** | 2.336 | **1.966** | 1.649 | 1.284 |
| 500 | **2.586** | 2.334 | **1.965** | 1.648 | 1.283 |
| 1000 | **2.581** | 2.330 | **1.962** | 1.646 | 1.282 |
| 2000 | **2.578** | 2.328 | **1.961** | 1.646 | 1.282 |
| Large | **2.576** | 2.326 | **1.960** | 1.645 | 1.282 |

# Formulas and Tables by Mario F. Triola



## Table A-4 Chi-Square ($\chi^2$) Distribution

| Degrees of Freedom | Area to the Right of the Critical Value | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.995 | 0.99 | 0.975 | 0.95 | 0.90 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 1 | — | — | 0.001 | 0.004 | 0.016 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.071 | 12.833 | 15.086 | 16.750 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.299 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| 17 | 5.697 | 6.408 | 7.564 | 8.672 | 10.085 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 |
| 19 | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| 21 | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 14.042 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| 23 | 9.260 | 10.196 | 11.689 | 13.091 | 14.848 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 |
| 24 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.559 |
| 25 | 10.520 | 11.524 | 13.120 | 14.611 | 16.473 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 |
| 26 | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 |
| 27 | 11.808 | 12.879 | 14.573 | 16.151 | 18.114 | 36.741 | 40.113 | 43.194 | 46.963 | 49.645 |
| 28 | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |
| 29 | 13.121 | 14.257 | 16.047 | 17.708 | 19.768 | 39.087 | 42.557 | 45.722 | 49.588 | 52.336 |
| 30 | 13.787 | 14.954 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |
| 40 | 20.707 | 22.164 | 24.433 | 26.509 | 29.051 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |
| 50 | 27.991 | 29.707 | 32.357 | 34.764 | 37.689 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 |
| 60 | 35.534 | 37.485 | 40.482 | 43.188 | 46.459 | 74.397 | 79.082 | 83.298 | 88.379 | 91.952 |
| 70 | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 |
| 80 | 51.172 | 53.540 | 57.153 | 60.391 | 64.278 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 |
| 90 | 59.196 | 61.754 | 65.647 | 69.126 | 73.291 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 |
| 100 | 67.328 | 70.065 | 74.222 | 77.929 | 82.358 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 |

*Source:* From Donald B. Owen, *Handbook of Statistical Tables.*

**Degrees of Freedom**

| | |
|---|---|
| $n - 1$ | Confidence Interval or Hypothesis Test with a standard deviation or variance |
| $k - 1$ | Goodness-of-Fit with $k$ categories |
| $(r - 1)(c - 1)$ | Contingency Table with $r$ rows and $c$ columns |
| $k - 1$ | Kruskal-Wallis test with $k$ samples |

# Formulas and Tables by Mario F. Triola



## Ch. 3: Descriptive Statistics

$\bar{x} = \dfrac{\Sigma x}{n}$ Mean

$\bar{x} = \dfrac{\Sigma(f \cdot x)}{\Sigma f}$ Mean (frequency table)

$s = \sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n - 1}}$ Standard deviation

$s = \sqrt{\dfrac{n(\Sigma x^2) - (\Sigma x)^2}{n(n - 1)}}$ Standard deviation (shortcut)

$s = \sqrt{\dfrac{n[\Sigma(f \cdot x^2)] - [\Sigma(f \cdot x)]^2}{n(n - 1)}}$ Standard deviation (frequency table)

variance $= s^2$

## Ch. 4: Probability

$P(A \text{ or } B) = P(A) + P(B)$ if $A$, $B$ are mutually exclusive

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ if $A$, $B$ are not mutually exclusive

$P(A \text{ and } B) = P(A) \cdot P(B)$ if $A$, $B$ are independent

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ if $A$, $B$ are dependent

$P(\bar{A}) = 1 - P(A)$ Rule of complements

${}_nP_r = \dfrac{n!}{(n - r)!}$ Permutations (no elements alike)

$\dfrac{n!}{n_1!\, n_2! \cdots n_k!}$ Permutations ($n_1$ alike, . . . )

${}_nC_r = \dfrac{n!}{(n - r)!\, r!}$ Combinations

## Ch. 5: Probability Distributions

$\mu = \Sigma[x \cdot P(x)]$ Mean (prob. dist.)

$\sigma = \sqrt{\Sigma[x^2 \cdot P(x)] - \mu^2}$ Standard deviation (prob. dist.)

$P(x) = \dfrac{n!}{(n - x)!\, x!} \cdot p^x \cdot q^{n-x}$ Binomial probability

$\mu = n \cdot p$ Mean (binomial)

$\sigma^2 = n \cdot p \cdot q$ Variance (binomial)

$\sigma = \sqrt{n \cdot p \cdot q}$ Standard deviation (binomial)

$P(x) = \dfrac{\mu^x \cdot e^{-\mu}}{x!}$ Poisson distribution where $e = 2.71828$

## Ch. 6: Normal Distribution

$z = \dfrac{x - \mu}{\sigma}$ or $\dfrac{x - \bar{x}}{s}$ Standard score

$\mu_{\bar{x}} = \mu$ Central limit theorem

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$ Central limit theorem (Standard error)

## Ch. 7: Confidence Intervals (one population)

$\hat{p} - E < p < \hat{p} + E$ Proportion

where $E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}}$

$\bar{x} - E < \mu < \bar{x} + E$ Mean

where $E = t_{\alpha/2}\dfrac{s}{\sqrt{n}}$ ($\sigma$ unknown)

or $E = z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}$ ($\sigma$ known)

$\dfrac{(n - 1)s^2}{\chi_R^2} < \sigma^2 < \dfrac{(n - 1)s^2}{\chi_L^2}$ Variance

## Ch. 7: Sample Size Determination

$n = \dfrac{[z_{\alpha/2}]^2 0.25}{E^2}$ Proportion

$n = \dfrac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2}$ Proportion ($\hat{p}$ and $\hat{q}$ are known)

$n = \left[\dfrac{z_{\alpha/2}\sigma}{E}\right]^2$ Mean

## Ch. 9: Confidence Intervals (two populations)

$(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$

where $E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1} + \dfrac{\hat{p}_2\hat{q}_2}{n_2}}$

$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$ (Indep.)

where $E = t_{\alpha/2}\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$ (df = smaller of $n_1 - 1$, $n_2 - 1$)

($\sigma_1$ and $\sigma_2$ unknown and not assumed equal)

$E = t_{\alpha/2}\sqrt{\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}}$ (df $= n_1 + n_2 - 2$)

$s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$

($\sigma_1$ and $\sigma_2$ unknown but assumed equal)

$E = z_{\alpha/2}\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

($\sigma_1$, $\sigma_2$ known)

$\bar{d} - E < \mu_d < \bar{d} + E$ (Matched pairs)

where $E = t_{\alpha/2}\dfrac{s_d}{\sqrt{n}}$ (df $= n - 1$)

# Formulas and Tables by Mario F. Triola



## Ch. 8: Test Statistics (one population)

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$ Proportion—one population

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$ Mean—one population ($\sigma$ unknown)

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$ Mean—one population ($\sigma$ known)

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$ Standard deviation or variance—one population

## Ch. 9: Test Statistics (two populations)

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\,\bar{q}}{n_1} + \frac{\bar{p}\,\bar{q}}{n_2}}}$$ Two proportions, $\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$ df = smaller of $n_1 - 1, n_2 - 1$

Two means—independent; $\sigma_1$ and $\sigma_2$ unknown, and not assumed equal.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$ (df $= n_1 + n_2 - 2$)

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Two means—independent; $\sigma_1$ and $\sigma_2$ unknown, but assumed equal.

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$ Two means—independent; $\sigma_1, \sigma_2$ known.

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$ Two means—matched pairs (df $= n - 1$)

$$F = \frac{s_1^2}{s_2^2}$$ Standard deviation or variance—two populations (where $s_1^2 \geq s_2^2$)

## Ch. 11: Goodness-of-Fit and Contingency Tables

$$\chi^2 = \Sigma\frac{(O - E)^2}{E}$$ Goodness-of-fit (df $= k - 1$)

$$\chi^2 = \Sigma\frac{(O - E)^2}{E}$$ Contingency table [df $= (r - 1)(c - 1)$]

$$\text{where } E = \frac{(\text{row total})(\text{column total})}{(\text{grand total})}$$

$$\chi^2 = \frac{(|b - c| - 1)^2}{b + c}$$ McNemar's test for matched pairs (df $= 1$)

## Ch. 10: Linear Correlation/Regression

$$\text{Correlation } r = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{n(\Sigma x^2) - (\Sigma x)^2}\sqrt{n(\Sigma y^2) - (\Sigma y)^2}}$$

$$\text{or } r = \frac{\sum (z_x z_y)}{n - 1}$$ where $z_x = z$ score for $x$, $z_y = z$ score for $y$

Slope: $$b_1 = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$$

$$\text{or } b_1 = r\frac{s_y}{s_x}$$

*y*-Intercept:

$$b_0 = \bar{y} - b_1\bar{x} \text{ or } b_0 = \frac{(\Sigma y)(\Sigma x^2) - (\Sigma x)(\Sigma xy)}{n(\Sigma x^2) - (\Sigma x)^2}$$

$\hat{y} = b_0 + b_1 x$ Estimated eq. of regression line

$$r^2 = \frac{\text{explained variation}}{\text{total variation}}$$

$$s_e = \sqrt{\frac{\Sigma(y - \hat{y})^2}{n - 2}} \text{ or } \sqrt{\frac{\Sigma y^2 - b_0\Sigma y - b_1\Sigma xy}{n - 2}}$$

$\hat{y} - E < y < \hat{y} + E$ Prediction interval

$$\text{where } E = t_{\alpha/2}s_e\sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\Sigma x^2) - (\Sigma x)^2}}$$

## Ch. 12: One-Way Analysis of Variance

Procedure for testing $H_0$: $\mu_1 = \mu_2 = \mu_3 = \cdots$

1. Use software or calculator to obtain results.
2. Identify the *P*-value.
3. Form conclusion:

   If *P*-value $\leq \alpha$, reject the null hypothesis of equal means.

   If *P*-value $> \alpha$, fail to reject the null hypothesis of equal means.

## Ch. 12: Two-Way Analysis of Variance

Procedure:

1. Use software or a calculator to obtain results.
2. Test $H_0$: There is no interaction between the row factor and column factor.
3. Stop if $H_0$ from Step 2 is rejected.

   If $H_0$ from Step 2 is not rejected (so there does not appear to be an interaction effect), proceed with these two tests:

   Test for effects from the row factor.

   Test for effects from the column factor.

# Formulas and Tables by Mario F. Triola



## Ch. 13: Nonparametric Tests

$$z = \frac{(x + 0.5) - (n/2)}{\frac{\sqrt{n}}{2}}$$ Sign test for $n > 25$

$$z = \frac{T - n(n+1)/4}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$ Wilcoxon signed ranks (matched pairs and $n > 30$)

$$z = \frac{R - \mu_R}{\sigma_R} = \frac{R - \frac{n_1(n_1 + n_2 + 1)}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}$$ Wilcoxon rank-sum (two independent samples)

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \cdots + \frac{R_k^2}{n_k}\right) - 3(N+1)$$

Kruskal-Wallis (chi-square df $= k - 1$)

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$ Rank correlation

$\left(\text{critical values for } n > 30\text{: } \frac{\pm z}{\sqrt{n-1}}\right)$

$$z = \frac{G - \mu_G}{\sigma_G} = \frac{G - \left(\frac{2n_1 n_2}{n_1 + n_2} + 1\right)}{\sqrt{\frac{(2n_1 n_2)(2n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}}}$$ Runs test for $n > 20$

## Ch. 14: Control Charts

$R$ chart: Plot sample ranges

UCL: $D_4\bar{R}$

Centerline: $\bar{R}$

LCL: $D_3\bar{R}$

$\bar{x}$ chart: Plot sample means

UCL: $\bar{\bar{x}} + A_2\bar{R}$

Centerline: $\bar{\bar{x}}$

LCL: $\bar{\bar{x}} - A_2\bar{R}$

$p$ chart: Plot sample proportions

UCL: $\bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n}}$

Centerline: $\bar{p}$

LCL: $\bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}}$

## Table A-6 Critical Values of the Pearson Correlation Coefficient $r$

| $n$ | $\alpha = .05$ | $\alpha = .01$ |
|---|---|---|
| 4 | .950 | .990 |
| 5 | .878 | .959 |
| 6 | .811 | .917 |
| 7 | .754 | .875 |
| 8 | .707 | .834 |
| 9 | .666 | .798 |
| 10 | .632 | .765 |
| 11 | .602 | .735 |
| 12 | .576 | .708 |
| 13 | .553 | .684 |
| 14 | .532 | .661 |
| 15 | .514 | .641 |
| 16 | .497 | .623 |
| 17 | .482 | .606 |
| 18 | .468 | .590 |
| 19 | .456 | .575 |
| 20 | .444 | .561 |
| 25 | .396 | .505 |
| 30 | .361 | .463 |
| 35 | .335 | .430 |
| 40 | .312 | .402 |
| 45 | .294 | .378 |
| 50 | .279 | .361 |
| 60 | .254 | .330 |
| 70 | .236 | .305 |
| 80 | .220 | .286 |
| 90 | .207 | .269 |
| 100 | .196 | .256 |

*NOTE:* To test $H_0: \rho = 0$ against $H_1: \rho \neq 0$, reject $H_0$ if the absolute value of $r$ is greater than the critical value in the table.

## Control Chart Constants

| Subgroup Size $n$ | $D_3$ | $D_4$ | $A_2$ |
|---|---|---|---|
| 2 | 0.000 | 3.267 | 1.880 |
| 3 | 0.000 | 2.574 | 1.023 |
| 4 | 0.000 | 2.282 | 0.729 |
| 5 | 0.000 | 2.114 | 0.577 |
| 6 | 0.000 | 2.004 | 0.483 |
| 7 | 0.076 | 1.924 | 0.419 |

# Overview of Statistical Methods

**Prepare**

1. **Context**
   What do the data mean?
   What is the goal of study?
2. **Source of the Data**
   Are the data from a source with a special interest so that there is pressure to obtain results that are favorable to the source?
3. **Sampling Method**
   Were the data collected in a way that is unbiased, or were the data collected in a way that is biased (such as a procedure in which respondents volunteer to participate)?

↓

**Analyze**

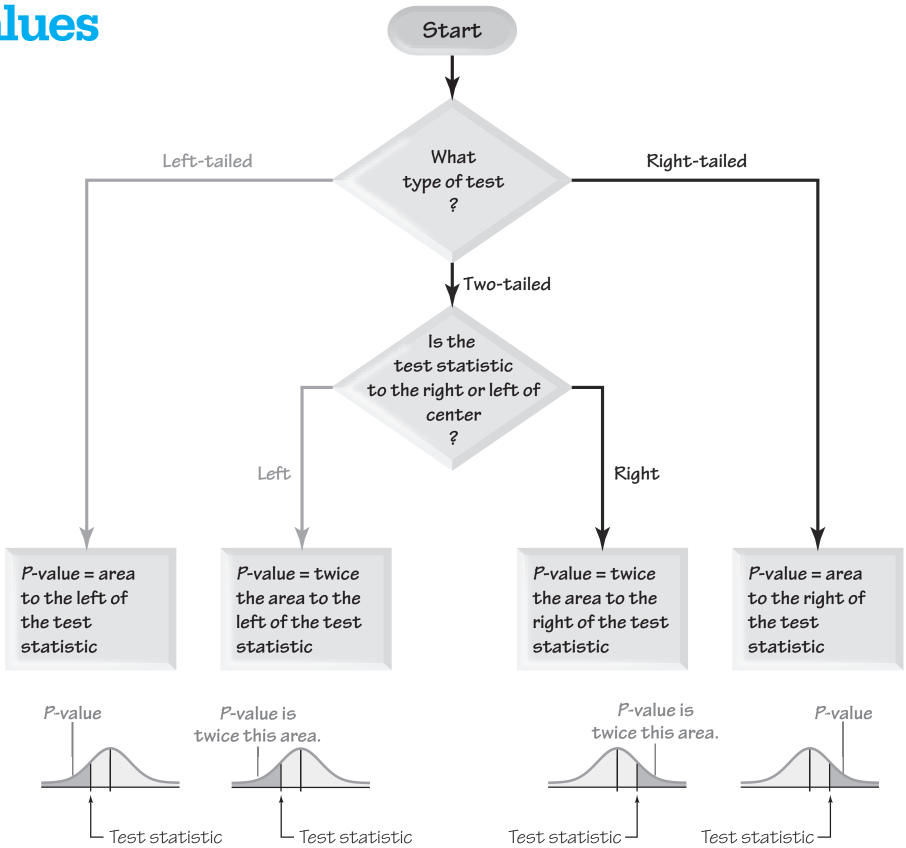
1. **Graph the Data**
2. **Explore the Data**
   Are there any outliers (numbers very far away from almost all of the other data)?
   What important statistics summarize the data (such as the mean and standard deviation)?
   How are the data distributed?
   Are there missing data?
   Did many selected subjects refuse to respond?
3. **Apply Statistical Methods**
   Use technology to obtain results.

↓

**Conclude**

1. **Statistical Significance**
   Do the results have statistical significance?
   Do the results have practical significance?

# Finding *P*-Values




| **Inferences about $\mu$: choosing between $t$ and normal distributions** | | |
|---|---|---|
| $t$ distribution: | | $\sigma$ not known and normally distributed population |
| | or | $\sigma$ not known and $n > 30$ |
| Normal distribution: | | $\sigma$ known and normally distributed population |
| | or | $\sigma$ known and $n > 30$ |
| Nonparametric method or bootstrapping: Population not normally distributed and $n \le 30$ | | |

# Symbol Table

| | |
|---|---|
| $\overline{A}$ | complement of event $A$ |
| $H_0$ | null hypothesis |
| $H_1$ | alternative hypothesis |
| $\alpha$ | alpha; probability of a type I error or the area of the critical region |
| $\beta$ | beta; probability of a type II error |
| $r$ | sample linear correlation coefficient |
| $\rho$ | rho; population linear correlation coefficient |
| $r^2$ | coefficient of determination |
| $R^2$ | multiple coefficient of determination |
| $r_s$ | Spearman's rank correlation coefficient |
| $b_1$ | point estimate of the slope of the regression line |
| $b_0$ | point estimate of the $y$-intercept of the regression line |
| $\hat{y}$ | predicted value of $y$ |
| $d$ | difference between two matched values |
| $\overline{d}$ | mean of the differences $d$ found from matched sample data |
| $s_d$ | standard deviation of the differences $d$ found from matched sample data |
| $s_e$ | standard error of estimate |
| $T$ | rank sum; used in the Wilcoxon signed-ranks test |
| $H$ | Kruskal-Wallis test statistic |
| $R$ | sum of the ranks for a sample; used in the Wilcoxon rank-sum test |
| $\mu_R$ | expected mean rank; used in the Wilcoxon rank-sum test |
| $\sigma_R$ | expected standard deviation of ranks; used in the Wilcoxon rank-sum test |
| $G$ | number of runs in runs test for randomness |
| $\mu_G$ | expected mean number of runs; used in runs test for randomness |
| $\sigma_G$ | expected standard deviation for the number of runs; used in runs test for randomness |
| $\mu_{\overline{x}}$ | mean of the population of all possible sample means $\overline{x}$ |
| $\sigma_{\overline{x}}$ | standard deviation of the population of all possible sample means $\overline{x}$ |
| $E$ | margin of error of the estimate of a population parameter, or expected value |
| $Q_1, Q_2, Q_3$ | quartiles |
| $D_1, D_2, \ldots, D_9$ | deciles |
| $P_1, P_2, \ldots, P_{99}$ | percentiles |
| $x$ | data value |

# Symbol Table

| | |
|---|---|
| $f$ | frequency with which a value occurs |
| $\Sigma$ | capital sigma; summation |
| $\Sigma x$ | sum of the values |
| $\Sigma x^2$ | sum of the squares of the values |
| $(\Sigma x)^2$ | square of the sum of all values |
| $\Sigma xy$ | sum of the products of each $x$ value multiplied by the corresponding $y$ value |
| $n$ | number of values in a sample |
| $n!$ | $n$ factorial |
| $N$ | number of values in a finite population; also used as the size of all samples combined |
| $k$ | number of samples or populations or categories |
| $\overline{x}$ | mean of the values in a sample |
| $\overline{R}$ | mean of the sample ranges |
| $\mu$ | mu; mean of all values in a population |
| $s$ | standard deviation of a set of sample values |
| $\sigma$ | lowercase sigma; standard deviation of all values in a population |
| $s^2$ | variance of a set of sample values |
| $\sigma^2$ | variance of all values in a population |
| $z$ | standard score |
| $z_{\alpha/2}$ | critical value of $z$ |
| $t$ | $t$ distribution |
| $t_{\alpha/2}$ | critical value of $t$ |
| df | number of degrees of freedom |
| $F$ | $F$ distribution |
| $\chi^2$ | chi-square distribution |
| $\chi^2_R$ | right-tailed critical value of chi-square |
| $\chi^2_L$ | left-tailed critical value of chi-square |
| $p$ | probability of an event or the population proportion |
| $q$ | probability or proportion equal to $1 - p$ |
| $\hat{p}$ | sample proportion |
| $\hat{q}$ | sample proportion equal to $1 - \hat{p}$ |
| $\overline{p}$ | proportion obtained by pooling two samples |
| $\overline{q}$ | proportion or probability equal to $1 - \overline{p}$ |
| $P(A)$ | probability of event $A$ |
| $P(A\|B)$ | probability of event $A$, assuming event $B$ has occurred |
| ${}_nP_r$ | number of permutations of $n$ items selected $r$ at a time |
| ${}_nC_r$ | number of combinations of $n$ items selected $r$ at a time |