A summary of definitions, postulates, algebra rules, and theorems that are often used in geometry proofs:

Definitions:

Definition of mid-point and segment bisector

If a line $BD$ intersects another line segment $AC$ at a point $M$ that makes $AM \cong MC$, then $M$ is the mid-point of segment $AC$, and $BD$ is a segment bisector of $AC$.

Definition of Adjacent Angles are two angles that share a common side with each other and have the same vertex.

In the above, $\angle ACB$ and $\angle BCD$ are adjacent angles, they share a common side $CB$ and have the same vertex, $C$.

Definition of Vertical Angles are two non-adjacent angles formed by two intersecting lines. Vertical angles also share the same vertex.

In the picture above, segment $AC$ intersects $BD$ at point $E$, so $\angle AED$ and $\angle BEC$ are vertical angles.
\(\angle BEA\) and \(\angle CED\) are also **vertical angles**.

**Definition of Right Angles and Perpendicular Lines:**
If two lines intersect and make the two adjacent angles equal to each other, then each of the equal angle is a **right angle**. The two lines that intersect this way is said to be **perpendicular** to each other.

![Diagram](image)

In the picture above, \(\overline{CA}\) intersects \(\overline{BD}\) at point \(E\) in such a way that makes \(\angle CEB \cong \angle AEB\). Therefore both \(\angle AEB\) and \(\angle CEB\) are both **right angles**.

Since they intersect to form right angles, segments \(\overline{CA}\) and \(\overline{BD}\) are **perpendicular** to each other. We write \(\overline{CA} \perp \overline{BD}\)

An **acute angle** is one which is less than a right angle.

an **obtuse angle** is one that is greater than a right angle.

![Diagram](image)

In the above, \(\angle ACD\) is acute, \(\angle ACB\) is right, and \(\angle ACE\) is obtuse.

In degree measure, a right angle has a measurement of 90°.

A straight line (straight angle) has a measurement of 180°

**Definition:** Two angles are **complementary** if, when placed adjacent to each other with one side in common, their non-common sides form a right angle. Numerically, we say that two angles are complementary if the sum of their degree measurement equals 90°

In the above picture, \(\angle ACD\) and \(\angle DCB\) are complementary because they form a right angle.
**Definition:** Two angles are **supplementary** if, when placed adjacent to each other with one side in common, their non-common sides form a straight line. Numerically, we say that two angles are supplementary if the sum of their degree measure equals 180°.

In the above, \( \angle ACD \) and \( \angle BCD \) are supplementary. Their non-common sides form the straight line \( BA \).

**Definition of Angle Bisector:**
If a line cuts an angle into two equal smaller angles, the line is said to **bisect** the angle and is an **angle bisector** of the angle.

In the picture above, \( \angle ACB \cong BCD \), so \( CB \) is the **angle bisector** of \( \angle ACD \).

A triangle where all three sides are unequal is a **scalene triangle**
A triangle where at least two of its sides is equal is an **isoceles triangle**
A triangle where all three sides are the same is an **equilateral triangle**.

A triangle where one of its angle is right is a **right triangle**.
In a right-triangle, the side that is opposite the right-angle is called the **hypotenuse** of the right-triangle. The other two sides are the **legs** of the right-triangle.

A triangle where one of its angle is obtuse is an **obtuse triangle**:
A triangle that does not have any obtuse angle (all three angles are acute) is called an **acute triangle**.

**Altitude of a Triangle**
In a triangle, if through any vertex of the triangle we draw a line that is perpen-
dicular to the side opposite the vertex, this line is an **altitude** of the triangle. The line opposite the vertex where the altitude is perpendicular to is the **base**.

\[
\text{In } \triangle ABC \text{ above, } \overline{BD} \text{ is an altitude. It contains vertex } B \text{ and is perpendicular to } \overline{AC}, \text{ which is the base.}
\]

**Median of a Triangle:**

In any triangle, if through one of its vertex we draw a line that **bisects** the opposite side, this line is called a **median** of the triangle.

\[
\text{In } \triangle ABC \text{ above, } \overline{BD} \text{ bisects } \overline{AC} \text{ in } D (\overline{AD} \cong \overline{DC}), \text{ so by definition, } \overline{BD} \text{ is a median of } \triangle ABC.
\]

**Angle Bisector**

An **angle bisector** of a triangle is a line that bisects an angle of the triangle and intersects the opposite side.

\[
\text{In } \triangle ABC \text{ above, } \overline{BD} \text{ is an angle bisector of } \angle ABC
\]
Properties, Postulates, Theorems:

Segment Addition Postulate:

In a line segment, if points $A, B, C$ are colinear and point $B$ is between point $A$ and point $C$, then: $AB + BC = AC$

Angle Addition Postulate:

The sum of the measure of two adjacent angles is equal to the measure of the angle formed by the non-common sides of the two adjacent angles.

In the above, $m\angle ACB + m\angle BCD = m\angle ACD$.

Properties of Equality:

For any object $x$, $x = x$ (reflexive property).

If $a = b$, then $b = a$ (symmetric property)

If $a = b$, and $b = c$, then $a = c$ (transitive property)

If $a = b$, then anywhere $a$ is used in a statement, $b$ can be used instead and the meaning of the statement is unchanged. (substitution property)

If $a = b$ and $c = d$, then $a + c = b + d$ (addition postulate)

If $a = b$ and $c = d$, then $a - c = b - d$ (subtraction postulate)

Complementary Angle Theorem: If two angles are complementary to the same angle, then they are congruent to each other

Supplementary Angle Theorem: If two angles are supplementary to the same angle, then they are congruent to each other

Vertical Angles Theorem: Vertical Angles are Congruent.

Ways to prove triangles are congruent:

Side-Side-Side (SSS)
If all three sides of a triangle is congruent to all three sides of another triangle, the two triangles are congruent.

Side-Angle-Side (SAS):
If two sides of a triangle is congruent to two sides of another triangle, and the angle formed by the two sides is also congruent, then the two triangles are congruent.

Angle-Side-Angle (ASA):
If two angles of a triangle is congruent to two angles of another triangle, and the side between the two angles is also congruent, then the two triangles are congruent.

Isoceles Triangle Theorem: In an isoceles triangle, the base angles (the angles on the opposite sides of the congruent sides) are congruent.

Equilateral Triangle Theorem: In an equilateral triangle, all three angles are congruent.