Optimization Problems:

1. A rectagular fence is to be made from a fixed perimeter, p. What is the dimension of the rectangle with the largest area that can be formed.

Ans: $\dfrac{p}{4}$

2. A rectangular container with an open top is to be made to have a volumn of 20 m^3. The length of its base must be twice of the width. Material for the base costs \$12 per square meter. Material for the sides costs \$9 per square meter. Find the dimension of the container that gives the cheapest cost.

Ans: $w = \left(\dfrac{45}{4}\right)^{1/3}$, $l = (90)^{1/3}$, $h = \dfrac{20}{3} \cdot \left(\dfrac{2}{75}\right)^{1/3}$

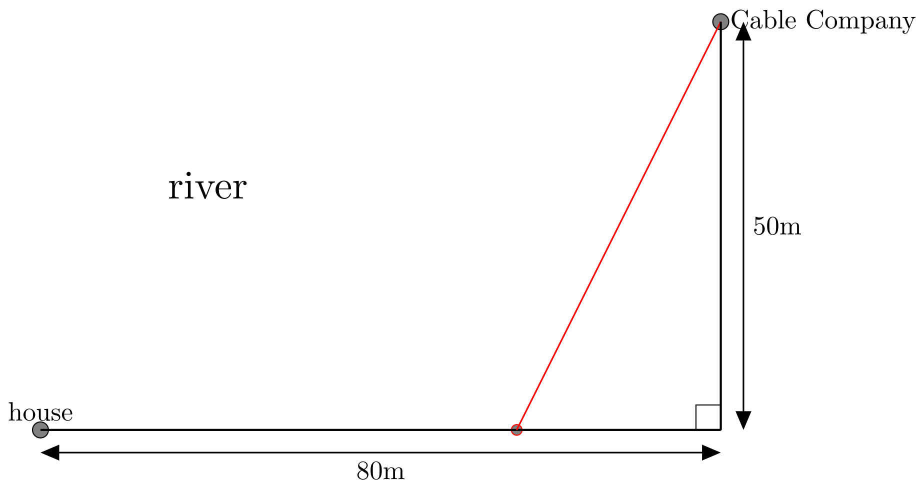
3. Find the point on the line $2x + y = 1$ that is closest to the point $(4, 6)$.

Ans: $\left(-\dfrac{6}{5}, \dfrac{17}{5}\right)$

4. A cylindrical can without a top is to be made to have a volumn of 100 m^3. Find the dimension of the can that minimizes the surface area.

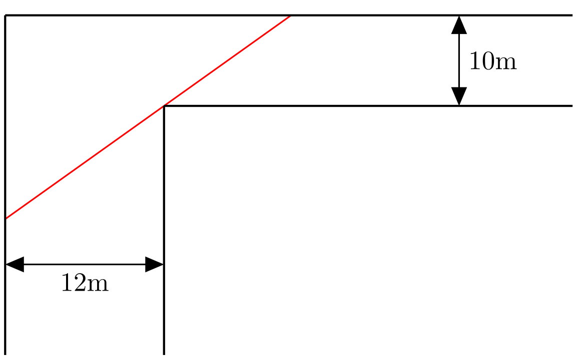
Ans: $\left(\dfrac{100}{\pi}\right)^{1/3}$

5. Cable wire is to be connected from the cable company to a house that is across a river and 80 meters downstream on the opposite bank. The river is 50 meters wide. If it costs \$10 per meter to install cable over water and \$7 per meter to install cable on land. How should the cable be installed so as to minimize cost?



Ans: The cable should be installed at a point p that is accross the river on the other side of the river, where the distance between p and the point directly accross the river from the company is given by x, and $x = \sqrt{\dfrac{122500}{51}}$,

6. A long and thin piece of steel is to be carried down a hallway that is 12 meters wide and make a right angle turn into another hallway that is 10 meters wide. What is the longest piece of steel that can be carried across?



Ans: $(10)^{2/3}\sqrt{(10)^{2/3} + (12)^{2/3}} + (12)^{2/3}\sqrt{(10)^{2/3} + (12)^{2/3}}$

7. A piece of wire 20 meters long is cut into two pieces. One piece is to form a square and another piece is to form a circle.

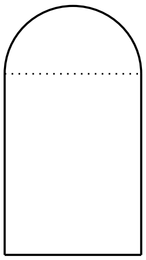
a. How should the piece be cut so that the total area is maximized?

Ans: Let x = length of the wire for square. Use $x = 0$. I.e. Use everything for the circle to maximize area.

b. How should the piece be cut so that the total area is minimized?

Ans: Let x = length of wire for square. Use $x = \dfrac{80}{\pi + 4}$

8. A norman window is made with a semi-circle on top of a rectangle. If the perimeter of the window is 10 meters long, what is the dimension of window so as to maximize the area?

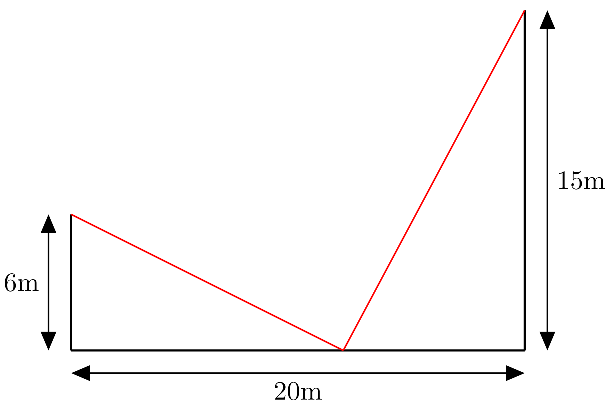


Ans: Let r = radius of circle. Use $r = \dfrac{10}{\pi + 4}$

9. What is the dimension of the rectangle with largest area that can be inscribed inside a circle of radius r?

Ans: Square, side = $r\sqrt{2}$

10. Two verticle poles, one 6 meters tall and another 15 meters tall, are separated by a distance of 20 meters. A point on the ground between the two poles is to be used to connect two wires to the top of the two poles.



a. Where should the point be pick so that the wire used is minimized?

Ans: Let x be the distance from the bottom of the shorter pole (the 6 meters pole) to the point where the wire is to be connected. Optimization occurs when $x = \dfrac{40}{7} \approx 5.71$ meters.

b. Where should the point be pick so that the angle formed with the two wires is maximized?

Ans: Same definition of x as in (a), use $x = \dfrac{-40 + \sqrt{4810}}{3} \approx 9.78$. Note: This part of the problem requires the derivative of the arctan function.