

Math 3E HW #8

Answers must be submitted on Moodle by 11AM on Tuesday, May 17th.

Good luck!

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Express the vector x as a linear combination of the u 's using Fourier's Trick.

1) $u_1 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}, u_3 = \begin{bmatrix} -2 \\ 6 \\ -4 \end{bmatrix}, x = \begin{bmatrix} 4 \\ -14 \\ 33 \end{bmatrix}$ 1) _____

A) $x = 5u_1 + 2u_2 - 4u_3$

B) $x = -5u_1 + 4u_2 + 4u_3$

C) $x = 10u_1 + 4u_2 - 8u_3$

D) $x = -5u_1 - 2u_2 + 4u_3$

Find the orthogonal projection of y onto u .

2) $y = \begin{bmatrix} -3 \\ -4 \end{bmatrix}, u = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$ 2) _____

A)

$$\begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

B)

$$\begin{bmatrix} 25 \\ -50 \end{bmatrix}$$

C)

$$\begin{bmatrix} \frac{1}{5} \\ -\frac{2}{5} \end{bmatrix}$$

D)

$$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Find the closest point to y in the subspace W spanned by u_1 and u_2 .

3) $y = \begin{bmatrix} 15 \\ 1 \\ 7 \end{bmatrix}, u_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, u_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ 3) _____

A)

$$\begin{bmatrix} 20 \\ 9 \\ 16 \end{bmatrix}$$

B)

$$\begin{bmatrix} 13 \\ 4 \\ 3 \end{bmatrix}$$

C)

$$\begin{bmatrix} 14 \\ 5 \\ 6 \end{bmatrix}$$

D)

$$\begin{bmatrix} -14 \\ -5 \\ -6 \end{bmatrix}$$

Find a QR factorization of the matrix A .

4) $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ 4) _____

A)

$$Q = \begin{bmatrix} 0 & \frac{3}{\sqrt{33}} & \frac{14}{\sqrt{330}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{33}} & \frac{2}{\sqrt{330}} \\ -\frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{33}} & \frac{9}{\sqrt{330}} \\ \frac{1}{\sqrt{3}} & -\frac{4}{\sqrt{33}} & \frac{7}{\sqrt{330}} \end{bmatrix}, R = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 11 & -3 \\ 0 & 0 & 30 \end{bmatrix}$$

B)

$$Q = \begin{bmatrix} 0 & 3 & 14 \\ 1 & 2 & 2 \\ -1 & -2 & 9 \\ 1 & -4 & 7 \end{bmatrix}, R = \begin{bmatrix} \frac{3}{\sqrt{3}} & 0 & 0 \\ \frac{1}{\sqrt{3}} & \frac{11}{\sqrt{33}} & 0 \\ 0 & -\frac{3}{\sqrt{33}} & \frac{30}{\sqrt{330}} \end{bmatrix}$$

C)

$$Q = \begin{bmatrix} 0 & 3 & 14 \\ 1 & 2 & 2 \\ -1 & -2 & 9 \\ 1 & -4 & 7 \end{bmatrix}, R = \begin{bmatrix} \frac{3}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & \frac{11}{\sqrt{33}} - \frac{3}{\sqrt{33}} & \\ 0 & 0 & \frac{30}{\sqrt{330}} \end{bmatrix}$$

D)

$$Q = \begin{bmatrix} 0 & \frac{3}{\sqrt{33}} & \frac{14}{\sqrt{330}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{33}} & \frac{2}{\sqrt{330}} \\ -\frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{33}} & \frac{9}{\sqrt{330}} \\ \frac{1}{\sqrt{3}} & -\frac{4}{\sqrt{33}} & \frac{7}{\sqrt{330}} \end{bmatrix}, R = \begin{bmatrix} \frac{3}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & \frac{11}{\sqrt{33}} - \frac{3}{\sqrt{33}} & \\ 0 & 0 & \frac{30}{\sqrt{330}} \end{bmatrix}$$

Find a least-squares solution of the inconsistent system $Ax = b$.

$$5) A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 9 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

5) _____

A)

$$\begin{bmatrix} -\frac{3}{4} \\ -\frac{1}{4} \end{bmatrix}$$

B)

$$\begin{bmatrix} \frac{37}{84} \\ -\frac{5}{21} \end{bmatrix}$$

C)

$$\begin{bmatrix} \frac{19}{18} \\ -\frac{7}{18} \end{bmatrix}$$

D)

$$\begin{bmatrix} \frac{57}{7016} \\ -\frac{21}{7016} \end{bmatrix}$$

Solve the problem.

6) Find the n th-order Fourier approximation to the function $f(t) = 4t$ on the interval $[0, 2\pi]$.

6) _____

A) $\pi - \cos(t) - \cos(2t) - \cos(3t) - \dots - \frac{8}{n}\cos(nt)$

B) $4\pi - 8\sin(t) - 4\sin(2t) - \frac{4}{3}\sin(3t) - \dots - \frac{4}{n}\sin(nt)$

C) $4\pi - 8\cos(t) - 4\sin(2t) - \frac{8}{3}\cos(3t) - \dots - \frac{8}{n}\cos(nt)$

D) $4\pi - 8\sin(t) - 4\sin(2t) - \frac{8}{3}\sin(3t) - \dots - \frac{8}{n}\sin(nt)$

Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits the given data points.

7) Data points: (2, 1), (3, 2), (7, 3), (8, 4)

7) _____

A) $y = \frac{5}{13} + \frac{4}{13}x$

B) $y = \frac{5}{13} + \frac{11}{26}x$

C) $y = -\frac{15}{13} + \frac{11}{26}x$

D) $y = -3 + \frac{11}{26}x$