Math 3E HW #7

Answers must be submitted on Moodle by 11AM on Thursday, May 5th.

Good luck!

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

For the given matrix A, find a basis for the corresponding eigenspace for the given eigenvalue.

1) $A = \begin{bmatrix} 1 & -4 & -4 \\ -4 & 1 & 4 \\ 4 & -4 & -7 \end{bmatrix}$, $\lambda = -3$ 1) _____

A) $\left\{ \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

B) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

C) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$

D) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

Find the eigenvalues of A, and find a basis for each eigenspace.

2) $A = \begin{bmatrix} 2 & 4 \\ -4 & 2 \end{bmatrix}$ 2) _____

A) $2 - 4i, \left\{ \begin{bmatrix} 1+2i \\ -4 \end{bmatrix} \right\}; 2 + 4i, \left\{ \begin{bmatrix} 1-2i \\ -4 \end{bmatrix} \right\}$

B) $2 - 4i, \left\{ \begin{bmatrix} 1-2i \\ -4 \end{bmatrix} \right\}; 2 + 4i, \left\{ \begin{bmatrix} 1+2i \\ -4 \end{bmatrix} \right\}$

C) $2 - 4i, \left\{ \begin{bmatrix} 1 \\ -i \end{bmatrix} \right\}; 2 + 4i, \left\{ \begin{bmatrix} 1 \\ i \end{bmatrix} \right\}$

D) $2 - 4i, \left\{ \begin{bmatrix} 1 \\ i \end{bmatrix} \right\}; 2 + 4i, \left\{ \begin{bmatrix} 1 \\ -i \end{bmatrix} \right\}$

The characteristic polynomial of a 5×5 matrix is given below. Find the eigenvalues and their multiplicities.

3) $\lambda^5 - 13\lambda^4 - 9\lambda^3 + 405\lambda^2$ 3) _____

A) 0 (multiplicity 1), 9 (multiplicity 3), −5 (multiplicity 1)
B) 0 (multiplicity 2), −9 (multiplicity 2), 5 (multiplicity 1)
C) 0 (multiplicity 2), 9 (multiplicity 2), −5 (multiplicity 1)
D) 0 (multiplicity 2), −9 (multiplicity 2), −5 (multiplicity 1)

Diagonalize the matrix A, if possible. That is, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

4) $A = \begin{bmatrix} 1 & 1 & 4 \\ 0 & -4 & 0 \\ -5 & -1 & -8 \end{bmatrix}$ 4) _____

A) $P = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -4 & 0 \\ 1 & 1 & 1 \end{bmatrix}, D = \begin{bmatrix} -4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

B) $P = \begin{bmatrix} 1 & -9 & -1 \\ -9 & -4 & 0 \\ 1 & -4 & 1 \end{bmatrix}, D = \begin{bmatrix} -4 & 1 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

C) $P = \begin{bmatrix} 1 & 0 & -1 \\ -9 & -4 & 0 \\ 1 & 1 & 1 \end{bmatrix}, D = \begin{bmatrix} -4 & 0 & -3 \\ 0 & -4 & 0 \\ 0 & -4 & -3 \end{bmatrix}$

D) $P = \begin{bmatrix} 1 & 0 & -1 \\ -9 & -4 & 0 \\ 1 & 1 & 1 \end{bmatrix}, D = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

5) $A = \begin{bmatrix} -3 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ -12 & 3 & -9 & 12 \\ 0 & 0 & 0 & -3 \end{bmatrix}$ 5) _____

A)

$$P = \begin{bmatrix} 4 & -2 & 1 & 0 \\ 8 & -2 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} -3 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -9 & 0 \\ 0 & 0 & 0 & -9 \end{bmatrix}$$

B) Not diagonalizable

C)

$$P = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ -2 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} -3 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -9 \end{bmatrix}$$

D)

$$P = \begin{bmatrix} 2 & 0 & -2 & 1 \\ 0 & 2 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} -3 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -9 \end{bmatrix}$$

Solve the initial value problem.

6) $dx/dt = A\mathbf{x}$, $x(0) = \begin{bmatrix} 3 \\ 3.2 \end{bmatrix}$, where $A = \begin{bmatrix} -4 & -3.125 \\ 8 & -4 \end{bmatrix}$ 6) _____

A) $\mathbf{x}(t) = \begin{bmatrix} -2\sin 5t + 3\cos 5t \\ 3.2\cos 5t + 4.8\sin 5t \end{bmatrix} e^{-4t}$

B) $\mathbf{x}(t) = \begin{bmatrix} 2\sin 5t + -3\cos 5t \\ -3.2\cos 5t - 4.8\sin 5t \end{bmatrix} e^{4t}$

C) $\mathbf{x}(t) = \begin{bmatrix} -2\sin 5t + 3\cos 5t \\ 3.2\cos 5t + 4.8\sin 5t \end{bmatrix} e^{4t}$

D) $\mathbf{x}(t) = \begin{bmatrix} 2\sin 5t - 3\cos 5t \\ -3.2\cos 5t - 4.8\sin 5t \end{bmatrix} e^{-4t}$