Math 3E HW #5

Answers must be submitted on Moodle by 11AM on Thursday, April 7th.

Good luck!

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Find all values of h such that $\mathbf{y}$ will be in the subspace of $\mathcal{R}^3$ spanned by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ if $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$, 1) ________

$\mathbf{v}_2 = \begin{bmatrix} 3 \\ 4 \\ -8 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} 7 \\ 7 \\ h \end{bmatrix}$.

A) $h = -14$ or 0
B) $h = -28$
C) $h = -14$
D) all $h \neq -14$

If the set W is a vector space, find a set S of vectors that spans it. Otherwise, state that W is not a vector space.

2) W is the set of all vectors of the form $\begin{bmatrix} a - 5b \\ 2 \\ 6a + b \\ -a - b \end{bmatrix}$, where a and b are arbitrary real numbers. 2) ________

A) $\begin{bmatrix} 1 \\ 0 \\ 6 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$

B) Not a vector space

C) $\begin{bmatrix} 1 \\ 0 \\ 6 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ 2 \\ 1 \\ -1 \end{bmatrix}$

D) $\begin{bmatrix} 1 \\ 2 \\ 6 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 1 \\ -1 \end{bmatrix}$

Find an explicit description of the null space of matrix A by listing vectors that span the null space.

3) $A = \begin{bmatrix} 1 & -2 & 3 & -3 & -1 \\ -2 & 5 & -5 & 4 & 2 \\ -1 & 3 & -2 & 1 & 1 \end{bmatrix}$ 3) ________

A) $\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

B) $\begin{bmatrix} -5 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

C) $\begin{bmatrix} 1 \\ 0 \\ 5 \\ -7 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ -2 \\ 0 \end{bmatrix}$

D) $\begin{bmatrix} -5 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Determine if the vector u is in the column space of matrix A and whether it is in the null space of A.

4) $\mathbf{u} = \begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix}$, $A = \begin{bmatrix} 1 & -3 & 4 \\ -1 & 0 & -5 \\ 3 & -3 & 6 \end{bmatrix}$ 4) ________

A) In Col A and in Nul A
B) In Col A, not in Nul A
C) Not in Col A, in Nul A
D) Not in Col A, not in Nul A

5) $\mathbf{u} = \begin{bmatrix} -4 \\ 3 \\ -4 \\ -2 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 0 & 3 \\ -2 & -1 & -4 \\ 3 & -3 & 0 \\ -1 & 3 & 6 \end{bmatrix}$ 5) ________

A) Not in Col A, not in Nul A
B) Not in Col A, in Nul A
C) In Col A and in Nul A
D) In Col A, not in Nul A

Find a basis for the column space of the matrix.

6) Let $A = \begin{bmatrix} -1 & 3 & 7 & 2 & 0 \\ 1 & -2 & -7 & -1 & 3 \\ 2 & -4 & -9 & -5 & 1 \\ 3 & -6 & -11 & -9 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 & -7 & -2 & 0 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 5 & -3 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. 6) ________

It can be shown that matrix A is row equivalent to matrix B. Find a basis for Col A.

A) $\begin{bmatrix} -1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -4 \\ -6 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -5 \\ -9 \end{bmatrix}$

B) $\begin{bmatrix} -1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -4 \\ -6 \end{bmatrix}, \begin{bmatrix} 7 \\ -7 \\ -9 \\ -11 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -5 \\ -9 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 1 \\ -1 \end{bmatrix}$

C) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 5 \\ 0 \end{bmatrix}$

D) $\begin{bmatrix} -1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -4 \\ -6 \end{bmatrix}, \begin{bmatrix} 7 \\ -7 \\ -9 \\ -11 \end{bmatrix}$

Determine whether the set of vectors is a basis for $\mathcal{R}^3$.

7) Given the set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$, decide which of the following statements is true: 7) ________

A: Set is linearly independent and spans $\mathcal{R}^3$. Set is a basis for $\mathcal{R}^3$.
B: Set is linearly independent but does not span $\mathcal{R}^3$. Set is not a basis for $\mathcal{R}^3$.
C: Set spans $\mathcal{R}^3$ but is not linearly independent. Set is not a basis for $\mathcal{R}^3$.
D: Set is not linearly independent and does not span $\mathcal{R}^3$. Set is not a basis for $\mathcal{R}^3$.

A) A
B) C
C) D
D) B

8) Given the set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$, decide which of the following statements is true: 8) ________

A: Set is linearly independent and spans $\mathcal{R}^3$. Set is a basis for $\mathcal{R}^3$.

B: Set is linearly independent but does not span $\mathcal{R}^3$. Set is not a basis for $\mathcal{R}^3$.

C: Set spans $\mathcal{R}^3$ but is not linearly independent. Set is not a basis for $\mathcal{R}^3$.

D: Set is not linearly independent and does not span $\mathcal{R}^3$. Set is not a basis for $\mathcal{R}^3$.

A) D B) C C) B D) A

Find the coordinate vector $[\mathbf{x}]_B$ of the vector x relative to the given basis B.

9) $\mathbf{b}_1 = \begin{bmatrix} 3 \\ 5 \\ -4 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 4 \\ -3 \\ -2 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 6 \\ -19 \\ 2 \end{bmatrix}$, and $B = \{\mathbf{b}_1, \mathbf{b}_2\}$ 9) ________

A) $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

B) $\begin{bmatrix} -2 \\ 2 \end{bmatrix}$

C) $\begin{bmatrix} -58 \\ 87 \\ 14 \end{bmatrix}$

D) $\begin{bmatrix} 42 \\ -38 \\ -12 \end{bmatrix}$