Math 3F Homework 1 (Fall 2017) Instructions: Please select the best solution among the options presented. Once you have made your selection for ALL problems below, enter your answers into the corresponding Moodle quiz no later than 9:30AM Thursday, August 31st. Good luck!

The Fitzhugh-Nagumo model for the electrical impulse in a neuron states that, in the absence of relaxation effects, the electrical potential in a neuron $v(t)$ obeys the differential equation

$$\frac{dv}{dt} = -v\left[v^2 - (1+a)v + a\right]$$

where a is a positive constant such that $0 < a < 1$.

Problem 1: What are the **equilibrium** solutions? Remember that equilibrium solutions correspond to solutions of the form $v(t) = \text{constant}$.

A **All** values in the interval $[-a, a]$.

B Just $v = 0$.

C There are no such solutions.

D $v = 1, v = a$.

E $v = 0, 1, a$.

Problem 2: Using the fact that this differential equation is separable, choose among the list below the implicit relationship between v and time t when v is not an equilibrium solution. Here, C is a constant and we have selected $a = \frac{1}{2}$.

A $\frac{v}{(1-2v)^3} = Ce^{-t}$.

B $\frac{v^2(1-v)^2}{(1-2v)^4} = Ce^{-t}$.

C $\frac{v\left(1-v^2\right)}{(1-2v)^4} = Ce^{-t}$.

D $\frac{v^2\left(1-v^2\right)}{(1-2v)^2} = Ce^{-2t}$.

Problem 3: Suppose that $v(0) = 2$. Use the quadratic formula to find $v(t)$ as an explicit function of time. Choose from among the list below the best possible answer.

A $v(t) = 2e^{-t}$.

B $v(t) = \frac{1-\frac{8}{9}e^{-t/2}-\sqrt{1-\frac{8}{9}e^{-t/2}}}{2\left(1-\frac{8}{9}e^{-t/2}\right)}$.

C $v(t) = 1 + e^{-t/2}$.

D $v(t) = \frac{1-\frac{8}{9}e^{-t/2}+\sqrt{1-\frac{8}{9}e^{-t/2}}}{2\left(1-\frac{8}{9}e^{-t/2}\right)}$.

According to Newton's Law of Universal Gravitation, the gravitational force on an object of mass m that has been projected vertically upward from the earth's surface is

$$F = \frac{mgR^2}{(x+R)^2}$$

where $x(t)$ is the object's distance above the surface at time t, R is the earth's radius, and g is the acceleration due to gravity. Also, by Newton's Second Law, $F = ma = m\frac{dv}{dt}$ and so

$$\frac{dv}{dt} = -\frac{gR^2}{(x+R)^2}$$

Suppose a rocket is fired vertically upward with an initial velocity v_0.

Problem 4: Using the Calculus II fact that

$$\frac{\frac{dv}{dt}}{\frac{dx}{dt}} = \frac{dv}{dx},$$

choose the expression from the list below that best represents $\frac{dv}{dx}$.

A $\frac{dv}{dx} = -\frac{v}{x}$

B $\frac{dv}{dx} = -\frac{gR^2}{v(x+R)^2}$

C $\frac{dv}{dx} = -\frac{2gR^2}{v^2(x+R)^2}$

D $\frac{dv}{dx} = -\frac{g^2v}{(x+R)^3}$

Problem 5: The result of problem 1 is a separable differential equation for the speed of the projectile as a function of height about the surface of the Earth. Solve this differential equation using the initial condition $v(x=0) = v_0$ and select the most appropriate solution from the list of functions below:

A $v(x) = \sqrt{\frac{2gR^2}{x+R} - 2gR + v_0^2}$

B $v(x) = \sqrt{\frac{2gR^2}{x+R} + 2gR}$

C $v(x) = \sqrt{\frac{2gR^2}{x+R} + gR - \frac{1}{2}v_0^2}$

D $v(x) = -\sqrt{\frac{2gR^2}{x+R} - 2gR + v_0^2}$

Problem 6: The escape velocity is the choice of v_0 so that $v(\infty) = 0$. Compute the escape velocity and select the answer from the list below

A $v_0 = \sqrt{gR^2}$

B $v_0 = \sqrt{2gR}$

C $v_0 = gR$

D $v_0 = \sqrt{4gR}$

Consider the chemical reaction whereby two reactant molecules A and B form the molecule C. According to the law of mass action, the differential equation for $x(t) = [C]$ (the concentration of C) is

$$\frac{dx}{dt} = k_f(a-x)(b-x) - k_r x$$

where k_f is the rate constant for the forward reaction and k_r is the rate constant for the reverse reaction. Both rate constants are positive. Here, the initial concentrations of A and B are a and b respectively and $x(0) = 0$. For simplicity, let's assume that $b = a = 1$ and $k_f = 1, k_r = 2$.

Problem 7: What are the **equilibrium** solutions?

A Just $2 + \sqrt{3}$.

B There are no such solutions.

C $x = 0, 2$.

D $x = 2 \pm \sqrt{3}$.

Problem 8: Find the solution to this equation with $x(0) = 0$.

A $x(t) = 2 - \frac{1}{t+1}$.

B $x(t) = 2 - \frac{1}{t+\frac{1}{2}}$.

C $x(t) = \frac{e^{2\sqrt{3}t}-1}{(2+\sqrt{3})e^{2\sqrt{3}t}+\sqrt{3}-2}$.

D $x(t) = \frac{e^{2\sqrt{3}t}-1}{(2-\sqrt{3})e^{2\sqrt{3}t}-\sqrt{3}-2}$.

Problem 9: Find the solution of $y'(x) = xe^y$ that satisfies $y(0) = 0$.

A $y(x) = -\ln(1 - \frac{1}{2}x^2)$.

B $y(x) = \sin(1 - \frac{1}{2}x^2)$.

C $y(x) = \ln(x^2 - 1)$.

D $y(x) = \ln(x^2 + 1)$.

Problem 10: Find the solution of $x \ln x = (1 + y^2)^{-1} y'$ that satisfies $y(0) = 0$ for $x \geq 0$.

A $y(x) = \cot\left(\frac{x^2}{4}(2\ln(x) - 1)\right)$.

B $y(x) = \exp\left(\frac{x^2}{4}(2\ln(x) - 1)\right)$.

C $y(x) = \tan\left(\frac{x^2}{4}(2\ln(x) - 1)\right)$.

D $y(x) = \tan^{-1}(x)$.