Math 3C  Homework #8

Due at 3:30PM on Thursday, April 27th 2017

Good luck!

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the volume of the indicated region.

1) the tetrahedron cut off from the first octant by the plane \( \frac{x}{8} + \frac{y}{7} + \frac{z}{4} = 1 \)
   
   A) 112  B) \( \frac{112}{3} \)  C) 56  D) \( \frac{224}{3} \)

2) the region bounded by the paraboloid \( z = 100 - x^2 - y^2 \) and the xy-plane
   
   A) \( \frac{10,000}{3} \pi \)  B) 5000 \( \pi \)  C) 500 \( \pi \)  D) \( \frac{1000}{3} \pi \)

3) the region bounded by the paraboloid \( z = 1 - \frac{x^2}{100} - \frac{y^2}{36} \) and the xy-plane
   
   A) 180 \( \pi \)  B) 20 \( \pi \)  C) 30 \( \pi \)  D) 300 \( \pi \)

4) the region bounded by the cylinder \( x^2 + y^2 = 9 \) and the planes \( z = 0 \) and \( x + z = 9 \)
   
   A) 27\( \pi \)  B) 81\( \pi \)  C) 729 \( \pi \)  D) 243\( \pi \)

5) the region in the first octant bounded by the coordinate planes and the surface \( z = 64 - x^2 - y \)
   
   A) 8192  B) \( \frac{32,768}{3} \)  C) \( \frac{65,536}{9} \)  D) \( \frac{131,072}{15} \)

6) the region in the first octant bounded by the coordinate planes and the planes \( x + z = 5 \), \( y + 6z = 30 \)
   
   A) 125  B) \( \frac{125}{6} \)  C) 625  D) 25

7) the region common to the interiors of the cylinders \( x^2 + y^2 = 36 \) and \( x^2 + z^2 = 36 \)
   
   A) 144  B) 864  C) 1152  D) 288

8) the region bounded by the coordinate planes and the planes \( z = x + y \), \( z = 6 \)
   
   A) 72  B) 108  C) 36  D) 54

9) the region bounded by the coordinate planes, the parabolic cylinder \( z = 9 - x^2 \), and the plane \( y = 4 \)
   
   A) 162  B) 81  C) 216  D) 72
10) the region bounded by the paraboloid \( z = x^2 + y^2 \) and the cylinder \( x^2 + y^2 = 100 \)

A) \( \frac{40000}{3\pi} \)  
B) \( \frac{10000}{3\pi} \)  
C) 5000\pi  
D) 15000\pi

Use the given transformation to evaluate the integral.

11) \( u = 2x + y - z, \ v = -x + y + z, \ w = -x + y + 2z; \)

\[
\int \int \int_{R} dx \ dy \ dz,
\]

where \( R \) is the parallelepiped bounded by the planes \( 2x + y - z = 2, 2x + y - z = 10, -x + y + z = 2, -x + y + z = 6, -x + y + 2z = 5, -x + y + 2z = 6 \)

A) 96  
B) \( \frac{32}{3} \)  
C) \( \frac{64}{3} \)  
D) 48

12) \( u = 2x + y - z, \ v = -x + y + z, \ w = -x + y + 2z; \)

\[
\int \int \int_{R} (2x + y - z)(z - x + y)(2z - x + y) \ dx \ dy \ dz,
\]

where \( R \) is the parallelepiped bounded by the planes \( 2x + y - z = 2, 2x + y - z = 3, -x + y + z = 7, -x + y + z = 9, -x + y + 2z = 2, -x + y + 2z = 9 \)

A) 2310  
B) 4620  
C) \( \frac{6160}{3} \)  
D) \( \frac{1540}{3} \)

Solve the problem.

13) Find the center of mass of the region of density \( \rho(x, y, z) = \frac{1}{4 - x^2 - y^2} \) bounded by the paraboloid \( z = 4 - x^2 - y^2 \) and the xy-plane.

A) \( \bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{1}{2} \)  
B) \( \bar{x} = 0, \bar{y} = 0, \bar{z} = 1 \)  
C) \( \bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{2}{3} \)  
D) \( \bar{x} = 0, \bar{y} = 0, \bar{z} = 2 \)

Use cylindrical coordinates to find the volume of the indicated region.

14) the region enclosed by the paraboloids \( z = x^2 + y^2 - 7 \) and \( z = 43 - x^2 - y^2 \)

A) 625\pi  
B) 1875\pi  
C) 1250\pi  
D) 2500\pi

15) the region enclosed by the cylinder \( x^2 + y^2 = 36 \) and the planes \( z = 0 \) and \( x + y + z = 12 \)

A) 864\pi  
B) 648\pi  
C) 432\pi  
D) 216\pi

16) the region enclosed by the paraboloids \( z = x^2 + y^2 - 2 \) and \( z = 30 - x^2 - y^2 \)

A) 1024\pi  
B) 256\pi  
C) 768\pi  
D) 512\pi
Use spherical coordinates to find the volume of the indicated region.

17) the region bounded above by the sphere \( x^2 + y^2 + z^2 = 64 \) and below by the cone \( z = \sqrt{x^2 + y^2} \)

A) \( 128\pi(2 - \sqrt{3}) \)  

B) \( \frac{512}{3}\pi(2 - \sqrt{2}) \)  

C) \( 128\pi(2 - \sqrt{2}) \)  

D) \( \frac{512}{3}\pi(2 - \sqrt{3}) \)  

18) the region enclosed by the cone \( z^2 = x^2 + y^2 \) between the planes \( z = 4 \) and \( z = 9 \)

A) \( \frac{665}{4} \)  

B) \( \frac{665}{3} \)  

C) \( \frac{665}{4}\pi \)  

D) \( \frac{665}{3}\pi \)  

19) the region that lies inside the sphere \( x^2 + y^2 + z^2 = 49 \) and outside the cylinder \( x^2 + y^2 = 36 \)

A) \( \frac{5(343 - 13^{3/2})\pi}{2} \)  

B) \( \frac{4(343 - 13^{3/2})\pi}{3} \)  

C) \( \frac{2(343 - 13^{3/2})\pi}{3} \)  

D) \( \frac{3(343 - 13^{3/2})\pi}{2} \)